Control of a Handwriting Robot with DOF-Redundancy based on Feedback in Task-Coordinates

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Abstract

In order to enhance dexterity of a robot hand, it is said that the robot should be designed to have a redundant number of degrees-of-freedom (DOF). However, in redundant robotic systems, the inverse kinematics from task-description space to joint space becomes ill-posed. Therefore the problem of determination of joint motions becomes difficult. In order to avoid this ill-posedness, many methods have been proposed, most of which introduce an additional input term calculated from some intentionally artificial index of performance. This paper treats a 4-DOF redundant handwriting robot by using a novel and simple control method for resolving the problem of such ill-posedness based on feedback in task-coordinates. The effectiveness of the proposed control method is demonstrated through computer simulation.

1 Introduction

In order to enhance dexterity of a robot hand, it is said that appropriate increase of the number of degrees-of-freedom (DOF) of the hand is indispensable. However, if the number of DOF of the robot dynamics increases more than the number of DOF in the task-description space, the inverse kinematics becomes ill-posed. Hence the problem of determination of joint motions for execution of given tasks becomes more difficult. Many methods proposed so far for a redundant manipulator to avoid this ill-posedness problem by using an additional input which is determined from minimization of some intentional index of performance[1]∼[4].

In this paper, we propose a simple control method for resolving this ill-posedness. This control method is called a Sensory Feedback Method and based on a transposed Jacobian matrix associated with position feedback on the basis of position data in task-coordinates measured by external sensors (such as a video camera and a laser range finder). The idea is grounded on the point of view of "sensory-motor coordination” that intelligence of human limb motions founds on as discussed not only in robotics but also in developmental psychology[5]. It is neither necessary to calculate the joint motions, nor necessary to calculate a control input in null-space by using a Jacobian pseudoinverse. This method is more natural and realistic than the conventional method depending on minimization of an intentionally introduced performance index together with the Computed Torque Method. The stability problem of this method cannot be proved with the aid of a Lyapunov function. Instead, this problem is solved by using a theorem of “stability on a manifold”[6]∼[8].

This paper treated typical robotic tasks of handwriting[9] with DOF redundancy by using a 4 DOF handwriting robot model whose endpoint is constrained on a two-dimensional plane(such as a sheet of paper). The effectiveness of the proposed control method is verified through computer simulation.

2 Modeling of Handwriting Robot

Two types of handwriting robot are treated in this paper. The first type is four-link planar robot, the second type is four-link three-dimensional (3D) robot. Each types of robot has a pen at the robot’s endpoint that moves on the XY-plane.

In following figures, \( x = [x, y] \) denotes a position of the robot’s endpoint is expressed in task-coordinates (Cartesian coordinates), \( x_d = [x_d, y_d] \) is a target position. Here, \( x \) can be always measurable with an external sensor.

2.1 Planar 4-DOF Robot Model

A planar 4-DOF handwriting robot as shown in Fig.1 can be described by the following Lagrange equation

\[
H(q)\ddot{q} + \left\{ \frac{1}{2} \dot{H}(q) + S(q,\dot{q}) \right\} \dot{q} = u \tag{1}
\]

where \( q = [q_1, q_2, q_3, q_4]^T \) denotes the vector of joint angles, \( H(q) \) is a \( 4 \times 4 \) inertia matrix, \( S(q,\dot{q}) \) is a
skew-symmetric matrix, \( u \) is a control input vector.

\[
\begin{pmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_4
\end{pmatrix} = \begin{pmatrix}
0 \\
l_2 \cos q_2 + l_3 \cos (q_2 + q_3) + l_4 \cos (q_2 + q_3 + q_4) \\
l_3 \cos (q_2 + q_3) + l_4 \cos (q_2 + q_3 + q_4) \\
l_4 \cos (q_2 + q_3 + q_4)
\end{pmatrix} ^T
\]

2.2 3D 4-DOF Robot Model

Robot dynamics of a 3D 4-DOF handwriting robot as shown in Fig.2 can be described by the following Lagrange equation with a holonomic constraint.

\[
H(q)\ddot{q} + \left\{ \frac{1}{2} \dot{H}(q) + S(q, \dot{q}) \right\} \dot{q} + \frac{\partial \phi(q)}{\partial q} ^T \lambda = u
\]

where \( q = [q_1, q_2, q_3, q_4]^T \) denotes the vector of joint angles, \( H(q) \) the 4 \times 4 inertia matrix, \( S(q, \dot{q}) \) a skew-symmetric matrix, \( g(q) \) the gravity term, \( \lambda \) a corresponding Lagrange multiplier (constraint force), \( u \) a control input vector. \( \phi(q) \) is the constraint that means the robot’s endpoint is always on a two-dimensional plane and described by

\[
\phi(q) = z \]

\[
= l_1 + l_2 \sin q_2 + l_3 \sin(q_2 + q_3) + l_4 \sin(q_2 + q_3 + q_4)
\]

\[
= 0
\]

Hence, the gradient of \( \phi(q) \) in \( q \) is described as

\[
\left( \frac{\partial \phi(q)}{\partial q} \right) ^T =
\begin{pmatrix}
0 \\
l_2 \cos q_2 + l_3 \cos (q_2 + q_3) + l_4 \cos (q_2 + q_3 + q_4) \\
l_3 \cos (q_2 + q_3) + l_4 \cos (q_2 + q_3 + q_4) \\
l_4 \cos (q_2 + q_3 + q_4)
\end{pmatrix}
\]

3 Simulation of Planar 4-D.O.F. Robot

3.1 Control Signal

We propose firstly a control signal which moves the endpoint \( x = [x, y]^T \) to a desired position \( x_d = [x_d, y_d]^T \). This is described as

\[
u = -C_1 \dot{q} - J_x ^T K_1 (x - x_d)
\]

where \( C_1 \) denotes a damping coefficient matrix for angular velocity feedback, \( K_1 \) a position feedback gain matrix from the measured position error in the XY-plane, \( J_x \) is Jacobian Matrix (2 \times 4) described as

\[
J_x = \begin{bmatrix}
\frac{\partial x}{\partial q} & \frac{\partial y}{\partial q}
\end{bmatrix}
\]

where \( (x, y) \) stands for the Cartesian coordinates in the XY-plane:

\[
\begin{align*}
x &= l_1 \cos q_1 + l_2 \cos(q_1 + q_2) + l_3 \cos(q_1 + q_2 + q_3) + l_4 \cos(q_1 + q_2 + q_3 + q_4) \\
y &= l_1 \sin q_1 + l_2 \sin(q_1 + q_2) + l_3 \sin(q_1 + q_2 + q_3) + l_4 \sin(q_1 + q_2 + q_3 + q_4)
\end{align*}
\]

Substituting eq.(5) into eq.(1) yields

\[
H(q)\ddot{q} + \left\{ \frac{1}{2} \dot{H}(q) + S(q, \dot{q}) + C_1 \right\} \dot{q} + J_x K_1 \Delta x = 0
\]

where \( \Delta x = x - x_d \). Taking inner product between \( \dot{q} \) and this equation yields

\[
\frac{d}{dt} E_1(q, \dot{q}, \Delta x) = -\dot{q}^T C_1 \dot{q}
\]
where $E_1(q, \dot{q}, \Delta x)$ is described as

$$E_1(q, \dot{q}, \Delta x) = \frac{1}{2} \dot{q}^T H(q) \dot{q} + \frac{1}{2} \Delta x^T K_1 \Delta x \tag{10}$$

If eq.(8) has an extra input $\Delta u$ in the right hand side that replaces zero, then taking inner product between eq.(8) and $\dot{q}$ yields

$$\int_0^t \dot{q} \Delta u = E_1(q, \dot{q}, \Delta x) - E_1(0) + \int_0^t \dot{q}^T C_1 \dot{q} \tag{11}$$

Hence,

$$\int_0^t \dot{q} \Delta u \geq -E_1(0) \tag{12}$$

This inequality shows that eq.(8) satisfies Passivity.

### 3.2 Simulation Parameter

Computer simulations based on physical parameters of the robot and feedback gain parameters as shown in Tables 1 and 2, and initial and desired conditions as shown in Table 3 and Table 4 will be presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>$l_1$</td>
<td>0.300</td>
<td>[m]</td>
</tr>
<tr>
<td>$m_1$</td>
<td>1.508</td>
<td>[kg]</td>
</tr>
<tr>
<td>$l_2$</td>
<td>4.584x10^-2</td>
<td>[kgm^2]</td>
</tr>
<tr>
<td>$m_2$</td>
<td>270</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_3$</td>
<td>766</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1.872x10^-2</td>
<td>[kgm^2]</td>
</tr>
<tr>
<td>$l_4$</td>
<td>100</td>
<td>[m]</td>
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<td>$m_4$</td>
<td>196</td>
<td>[kg]</td>
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<td>$l_5$</td>
<td>6.851x10^-4</td>
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<td>$m_5$</td>
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<td>$l_6$</td>
<td>0.031</td>
<td>[kg]</td>
</tr>
<tr>
<td>$l_7$</td>
<td>1.055x10^-4</td>
<td>[kgm^2]</td>
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### 3.3 Simulation Results

A better damping gain $C$ is chosen firstly after several trials of simulation by changing values of $C$ together with $K$. Then, transient responses of $x, y$ are shown in Figs. 3 and 4 with changing the target position $x_d$ (Table 3). Every choice for gains of Table 2 attains convergence to the target position.
A desired force \( \lambda \) which moves the endpoint joint actuators. We propose secondly a control signal exactly, or can be computed by optical encoders of \( J \) measured position error in the XY-plane.

### 4.1 Control Signal

And the trajectory of the robot’s endpoint is shown in Fig. 5.

![Fig. 4: Response of x](image)

![Fig. 4: Response of y](image)

![Fig. 5: Trajectory of endpoint](image)

### 4 Simulation of Planar 4-D.O.F. Robot

#### 4.1 Control Signal

It is assumed that the gravity term \( g(q) \) is known exactly, or can be computed by optical encoders of joint actuators. We propose secondly a control signal which moves the endpoint \( x = [x, y]^T \) to target position \( x_d = [x_d, y_d]^T \), and brings the pressing force \( \lambda \) to a desired force \( \lambda_d > 0 \). This is described as

\[
\mathbf{u} = g(q) - C_2 \dot{q} - J_x^T K_2 (x - x_d) - \left( \frac{\partial \phi}{\partial \mathbf{q}} \right)^T \lambda_d \tag{13}
\]

where \( g(q) \) denotes the gravity compensation, \( C_2 \) a damping coefficient matrix for angular velocity feedback, \( K_2 \) a position feedback gain matrix from the measured position error in the XY-plane, \( J_x \) is Jacobian Matrix \( (2 \times 4) \) described as

\[
J_x = \begin{bmatrix}
\frac{\partial x}{\partial \mathbf{q}} \\
\frac{\partial y}{\partial \mathbf{q}}
\end{bmatrix} \tag{14}
\]

where \( (x, y) \) is kinematics of this robot

\[
\begin{align*}
\dot{x} &= \cos q_1 \left\{ l_2 \cos q_2 + l_3 \cos(q_2 + q_3) \\
&+ l_4 \cos(q_2 + q_3 + q_4) \right\} \\
\dot{y} &= \sin q_1 \left\{ l_2 \sin q_2 + l_3 \sin(q_2 + q_3) \\
&+ l_4 \sin(q_2 + q_3 + q_4) \right\}
\end{align*} \tag{15}
\]

Substituting eq.\( (13) \) into eq.\( (2) \) yields

\[
H(q) \ddot{q} + \left\{ \frac{1}{2} \dot{H}(q) + S(q, \dot{q}) + C_2 \right\} \dot{q} + J_x K_2 \Delta x - \left( \frac{\partial \phi(q)}{\partial \mathbf{q}} \right)^T \Delta \lambda = 0 \tag{16}
\]

where \( \Delta x = x - x_d \) and \( \Delta \lambda = \lambda - \lambda_d \). Taking inner product between \( \dot{q} \) and this equation yields

\[
\frac{d}{dt} E_2(q, \dot{q}, \Delta x) = -\dot{q}^T C_2 \dot{q} \tag{17}
\]

where \( E_2(q, \dot{q}, \Delta x) \) is described as

\[
E_2(q, \dot{q}, \Delta x) = \frac{1}{2} \dot{q}^T H(q) \dot{q} + \frac{1}{2} \Delta x^T K_2 \Delta x \tag{18}
\]

Taking inner product between \( \dot{q} \) and eq.\( (16) \) if zero of the right hand side of eq.\( (16) \) is replaced with a new input \( \Delta u \) yields

\[
\int_0^t \dot{q} \Delta u = E_2(q, \dot{q}, \Delta x) - E_2(0) + \int_0^t \dot{q}^T C_2 \dot{q} \tag{19}
\]

Hence,

\[
\int_0^t \dot{q} \Delta u \geq -E_2(0) \tag{20}
\]

This inequality shows that eq.\( (16) \) satisfies Passivity.

#### 4.2 Simulation Parameter

Computer simulations are based on physical parameters of the robot and feedback gain parameters are shown in Tables 5 and 6, and initial and desired conditions are shown in Table 7 and Table 8.
Table 5: Link parameters

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>0.030</td>
<td>[m]</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>24.09 \times 10^{-3}</td>
<td>[kg]</td>
</tr>
</tbody>
</table>
| \( I_1 \) | \[
\begin{bmatrix}
0.201 & 0 & 0 \\
0 & 0.040 & 0 \\
0 & 0 & 0.201
\end{bmatrix}
\times 10^{-5} | [kgm^2] |
| \( l_2 \) | 0.050 | [m]  |
| \( m_2 \) | 40.15 \times 10^{-3} | [kg] |
| \( I_2 \) | \[
\begin{bmatrix}
0.067 & 0 & 0 \\
0 & 0.870 & 0 \\
0 & 0 & 0.870
\end{bmatrix}
\times 10^{-5} | [kgm^2] |
| \( l_3 \) | 0.050 | [m]  |
| \( m_3 \) | 40.15 \times 10^{-3} | [kg] |
| \( I_3 \) | \[
\begin{bmatrix}
0.067 & 0 & 0 \\
0 & 0.870 & 0 \\
0 & 0 & 0.870
\end{bmatrix}
\times 10^{-5} | [kgm^2] |
| \( l_4 \) | 0.030 | [m]  |
| \( m_4 \) | 24.09 \times 10^{-3} | [kg] |
| \( I_4 \) | \[
\begin{bmatrix}
0.040 & 0 & 0 \\
0 & 0.201 & 0 \\
0 & 0 & 0.201
\end{bmatrix}
\times 10^{-5} | [kgm^2] |

Table 6: Choice for gains

<table>
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<th>Constant</th>
<th>Value</th>
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<tbody>
<tr>
<td>( K_2 ) [N/m]</td>
<td>150</td>
</tr>
<tr>
<td>( C_2 ) [smN]</td>
<td></td>
</tr>
<tr>
<td>&amp; 020000</td>
<td></td>
</tr>
<tr>
<td>&amp; 0.02 0.02 0.02</td>
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<tr>
<td>&amp; 0000 0002</td>
<td></td>
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<td>&amp; 0000 0.02</td>
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</table>

Table 7: Initial conditions

<table>
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<tr>
<th>Parameter</th>
<th>Position</th>
<th>Force</th>
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</thead>
<tbody>
<tr>
<td>Variable</td>
<td>( x_0 )</td>
<td>( y_0 )</td>
</tr>
<tr>
<td>Unit</td>
<td>[m]</td>
<td>[m]</td>
</tr>
<tr>
<td>1st time</td>
<td>0</td>
<td>5.00 \times 10^{-2}</td>
</tr>
<tr>
<td>2nd time</td>
<td>0</td>
<td>1.00 \times 10^{-1}</td>
</tr>
<tr>
<td>3rd time</td>
<td>7.59 \times 10^{-2}</td>
<td>7.59 \times 10^{-2}</td>
</tr>
<tr>
<td>4th time</td>
<td>-3.54 \times 10^{-2}</td>
<td>3.54 \times 10^{-2}</td>
</tr>
</tbody>
</table>

Table 8: Desired conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
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</thead>
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<tr>
<td>Desired position</td>
<td>( x_d )</td>
<td>0.00</td>
<td>[m]</td>
</tr>
<tr>
<td>Desired</td>
<td>( y_d )</td>
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<td>[m]</td>
</tr>
<tr>
<td>Desired force</td>
<td>( \lambda_d )</td>
<td>0.50</td>
<td>[N]</td>
</tr>
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</table>

4.3 Simulation Results

Numerical solutions of the differential equations under the geometric constraint are obtained by using Baumgarte’s method [10], in which a coefficient \( \gamma_f \) for a highly over-damped second order differential equation corresponding to the constraint \( z = 0 \), that is,

\[
\ddot{Q} + \gamma f \dot{Q} + \frac{\gamma_f^2}{4} Q = 0
\]

is chosen as \( \gamma_f = 1000 \).

In this simulation, a better choice for the damping gain \( C = cI \) with \( c = 0.02 \) is chosen firstly after several trials of simulation by changing values of \( c \) together with \( K \). Then, transient responses of \( x, y \) and constraint force \( \lambda \) are shown in Figs. 6 to 8 with changing the initial position \( \mathbf{x}(0) \) (Table 7). Every choice shows convergence to the target value.

![Fig. 6: Responses of x](image)

![Fig. 7: Responses of y](image)

![Fig. 8: Responses of constraint force \( \lambda \)](image)

For reference, a result of trajectory tracking is shown Fig.9.

![Fig. 9: Trajectory tracking](image)
5 Conclusion

A novel method of redundancy resolution for robot tasks is proposed on the basis of a feedback method in task-coordinates through discussing an illustrative example of handwriting robots. Simulation results verify the validity of the proposed method and show that a human-like robot motion can be realized in the case of handwriting robot tasks without introducing any extra performance index.

These methods are apparently the same as PD feedback control in task coordinates. In order to prove the stability problem of this control law with the aid of a Lyapunov function, the dimension of work space and the dimension of joint space must be equal. However, in case of a redundant system, they are not equal. Hence the stability problem of this sensory-feedback method is not trivial and therefore it must be carefully treated by using a theorem of "stability on a manifold"[7]. A rough sketch of the proof for the case of sensory feedback of eq.(13) is given in Appendix.

References


Appendix

Note that $E_2$ eq.(18) is non-negative definite and its time derivative $\dot{E}_2$ described as

$$\dot{E}_2 = \frac{d}{dt} E_2 = -\dot{\mathbf{q}}^T C_2 \dot{\mathbf{q}} \quad (A-1)$$

is non-positive definite. However, $E_2$ is neither positive definite with respect to the state vector $(x, \dot{x})$ nor positive definite under the constraint manifold

$$M_\phi = \left\{ (x, \dot{x}) : \phi(x) = 0, \left(\frac{\partial \phi}{\partial \mathbf{q}}\right) \dot{\mathbf{q}} = 0 \right\} \quad (A-2)$$

because $E_2$ includes only a quadratic term of two-dimensional position-error variables $\Delta \mathbf{x}$. Therefore, the scalar quantity $E_2$ can not play a role of Lyapunov’s function for proving the stability of the closed-loop dynamics of eq.(16). It is therefore important to introduce a modified scalar quantity defined as

$$V = E_2 - \alpha \dot{\mathbf{q}} H(\mathbf{q}) P_\phi J_\phi^T \Delta \mathbf{x} \quad (A-3)$$

where $\alpha > 0$ is a constant specified later and $P_\phi$ is defined as

$$
\begin{align*}
P_\phi &= I_4 - J_\phi^+ J_\phi \\
J_\phi &= \left(\frac{\partial \phi}{\partial \mathbf{q}}\right) \\
J_\phi^+ &= J_\phi^T (J_\phi P_\phi J_\phi^T)^{-1}
\end{align*}
$$

Note that $P_\phi J_\phi^T = P_\phi \left(\frac{\partial \phi}{\partial \mathbf{q}}\right)^T = 0$. Hence, it follows that

$$\dot{V} = \dot{E}_2 - \alpha \dot{\mathbf{q}} H(\mathbf{q}) P_\phi J_\phi^T \Delta \mathbf{x}$$

$$- \left(\frac{1}{2} \dot{\mathbf{H}}(\mathbf{q}) P_\phi + H(\mathbf{q}) P_\phi \right) J_\phi^T \Delta \mathbf{x}$$

$$+ H(\mathbf{q}) P_\phi J_\phi^T \Delta \mathbf{x}$$

$$\leq \dot{E}_2 - \alpha k \Delta \mathbf{x}^T J_\phi P_\phi J_\phi^T \Delta \mathbf{x}$$

$$+ \frac{\alpha k^{-1}}{2} \mathbf{q}^T \mathbf{q}^2 + \frac{\alpha k^2}{2} \Delta \mathbf{x}^T J_\phi P_\phi J_\phi^T \Delta \mathbf{x}$$

$$- \dot{\mathbf{q}} \left\{ \left(\frac{1}{2} \dot{\mathbf{H}}(\mathbf{q}) P_\phi + S(\mathbf{q}, \dot{\mathbf{q}}) P_\phi + H(\mathbf{q}) \dot{P}_\phi \right) J_\phi^T \Delta \mathbf{x} \\
+ H(\mathbf{q}) P_\phi J_\phi^T \Delta \mathbf{x} + H(\mathbf{q}) P_\phi J_\phi^T J_\phi \dot{\mathbf{q}} \right\}$$
\[
\dot{V} \leq - \left( 1 - \frac{\alpha c}{2k} \right) c||\dot{q}||^2 - \frac{\alpha k}{2} \Delta x^T J_x \dot{P}_0 J_x^T \Delta x + O(10^{-6})||\dot{q}||^2 \quad (A-4)
\]

where \( ||\dot{q}||^2 = ||\dot{q}_1||^2 + ||\dot{q}_2||^2 \) and it is assumed that \( C = cI_4 \) and \( K = kI_2 \) and \( \lambda_M \) is of \( O(10^{-5}) \) as in Table 1. Since \( c \) is chosen sufficiently large in comparison with \( h_M \) and \( c/k = O(10^{-3}) \) it follows that

\[
\dot{V} \leq - \frac{c}{2} ||\dot{q}||^2 - \frac{\alpha\sigma_m}{2} k||\Delta x||^2 \quad (A-5)
\]

where

\[
\sigma_m = \inf_{t \in [0, \infty)} \{ \lambda_m \left( J_x \dot{P}_0 J_x^T \right) \} \quad (A-6)
\]

and \( \lambda_m \left( J_x \dot{P}_0 J_x^T \right) \) stands for the minimum eigenvalue of \( J_x \dot{P}_0 J_x^T \) for all \( q_1 \) and \( q_2 \), provided that \( \alpha \leq O(10^2) \). It is also important to note that

\[
V \geq E_2 - \frac{\alpha}{2} \lambda_M \dot{q}^T H(q) \dot{q} - \frac{\alpha}{2} \sigma_M ||\Delta x||^2 \quad (A-7)
\]

where

\[
\sigma_M = \sup_{t \in [0, \infty)} \{ \lambda_M \left( J_x \dot{P}_0 J_x^T \right) \} \quad (A-8)
\]

and \( \lambda_M \left( J_x \dot{P}_0 J_x^T \right) \) denotes the maximum eigenvalue of \( J_x \dot{P}_0 J_x^T \) over all \( q_1 \) and \( q_2 \). Then, it follows that

\[
V \geq (1 - \alpha\sigma_0) E_2 \quad (A-9)
\]

where

\[
\sigma_0 = \max \{ \lambda_M, \sigma_M/k \} \quad (A-10)
\]

and, at the same time,

\[
V \leq (1 + \alpha\sigma_0) E_2 \quad (A-11)
\]

According to Tables 1 and 3, \( \sigma_0 \) must be around in the order of \( 10^{-3} \). Hence, as far as \( \alpha \leq 10^2 \), \( 1 - \alpha\sigma_0 > 1/2 \) and hence \( V \) is nonnegative definite. Thus,

\[
(1 - \alpha\sigma_0) E_2 \leq V \leq (1 + \alpha\sigma_0) E_2 \quad (A-12)
\]

and it follows from eq.(A-5) that

\[
\dot{V} \leq -\rho E_2 \quad (A-13)
\]

where

\[
\rho = \min \left\{ \frac{c}{\lambda_M}, \alpha\sigma_m \right\} \quad (A-14)
\]

If \( \sigma_m \) is bounded from below by a constant \( \bar{\sigma} \) in a neighborhood of the initial state, then \( \rho \) can be chosen as \( \rho = \alpha\bar{\sigma} \) (note that \( \alpha\sigma_m \) is less than \( c/\lambda_M \) in ordinary cases because \( \alpha \leq 10^2 \) and \( \sigma_m \) is at most of \( O(10^{-2}) \)). Thus, it is concluded that \( V \) and \( E_2 \) converge to zero exponentially and all angular velocities \( \dot{q}_i \) and \( \Delta x \) converge to zero exponentially, too.

Now it is necessary to show that, during motion of the robot, all \( q_i \) remain in a neighbourhood of the initial state so that both the Jacobian vector \( J_\phi(q)^T \) and another Jacobian matrix \( J_x(q) \) are non-degenerated. This will be shown in a similar way to the argument presented in Appendix of the previous paper[7].