Rotational Contact Model of Soft Fingertip for Tactile Sensing
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Abstract—This paper proposes a new contact model of a soft fingertip attached on a rotational finger of one degree-of-freedom. We analyze the deformation of the soft fingertip statically, and introduce a virtual spring as an infinitesimal component of the soft fingertip. We correlate the derived force equation with a pressure distribution acting on a sensing plane. Finally we compare simulation results and experimental results to demonstrate our theoretical pressure model.

Index Terms—Soft fingertip, Tactile sensor, Pressure distribution, Rotational contact model

I. INTRODUCTION

Human fingers have a characteristic of viscoelasticity and deform easily according to the shape of an grasped object, and it is said that humans use a pressure information acts on a fingertip for grasping and manipulation. Furthermore, we can stably grasp and manipulate an object by using a plane contact between human fingertip and the object. In terms of above, recently many researches associated with the grasping using a soft fingertip have been studied. Nguyen et. al. proposed a simple deformation model of a soft fingertip in order to use analytical mechanics theory in control. But the deformation model described in that paper assume that all the elastic forces exerted in the soft fingertip face toward the origin of the fingertip. Therefore, that model can not be converted to a pressure model at all. Shimoga et. al. conducted an impact experiment so as to decide a best material for the soft fingertip. This paper concluded that a gel tip is most suitable for the soft fingertip by evaluating both of impact and strain energy dissipations quantitatively. Xydas et. al. proposed a power law in which a sectional area of the soft fingertip is associated with an normal force caused by normal deformation of the soft fingertip. The Von Mises stress and the strain tensor were introduced for derivation of the above law. But these techniques as theory of plasticity are so difficult and complicated to understand precisely. Our objective is not derivation of an exact model of the soft contact, but also a more simpler static model.

Inoue et al. has proposed a prismatic contact model between a soft fingertip and a rigid object. In this paper, we will propose a new rotational contact model between a soft fingertip and an object so as to use this model in tactile sensing for object manipulation. We introduce virtual springs into the soft fingertip, and derive a force equation caused by elastic force inside a soft material. We convert the force equation to a pressure equation by using an incompressibility of that material. Finally, we validate the pressure equation by comparing a simulation result and experimental result including rotation of a finger and rolling of an object.

II. CONTACT DEFORMATION MODEL OF SOFT FINGERTIP WITH ROTATIONAL JOINT

A. Concept

We find that a stable grasping and posture control of an object can be performed by a pair of 1 D.O.F rotational fingers with soft fingertips as shown in Fig.1. We have previously derived a simple prismatic contact model. In the previous paper, we have restricted a contact pattern to a prismatic contact shown in Fig.2-(a). In this paper we will define a rotational contact illustrated in Fig.2-(b). Note that prismatic contact is a subset of rotational contact.

Fig. 1. Implementation of manipulation task

Fig. 2. Grasping

(a) Prismatic contact
(b) Rotational contact

In prismatic contact of 2 D.O.F hand illustrated in Fig.2-(a), only position control of the object can be realized by moving along prismatic direction. On the other hand, only posture control of the object can be achieved in rotational contact of 2
D.O.F. hand illustrated in Fig.2-(b). Thus, when each contact pattern varies, both feasible tasks in control are different each other even if both hands have same 2 D.O.F. hand. Therefore, it is important to derive a rotational contact model of the soft fingertip attached to 1 D.O.F. rotational finger.

B. Formulation of Rotational Contact

We will describe a rotational contact model in this section. In previous paper, we have derived following equations:

\[ dF = E(1 - g(x, y))g(x, y)dS', \]  

where \( g(x, y) = \frac{r - d - x \cdot \sin \theta_{obj}}{\cos \theta_{obj} \sqrt{r^2 - (x^2 + y^2)}}. \)

\[ E \] is Young’s modulus of the material of the soft fingertip, \( r \) and \( d \) denote a radius and a maximum displacement of the soft fingertip respectively, and \( \theta_{obj} \) represents the orientation angle of the object, as shown in Fig.3. The coordinate of a sensing plane is denoted by \( (x, y) \). In this paper, we extend the proposed prismatic contact model to a rotational contact model by introducing a virtual spring into the soft fingertip as well as the prismatic contact model. In prismatic contact of 1 D.O.F. finger, all the infinitesimal forces act along the same direction as shown in Fig.3.

As shown in Fig.4, let \( L \) be the length of a finger, \( r \) be the radius of the soft fingertip, \( \theta_1 \) be the rotational angle of the finger, \( O \) be the center of the finger coordinate, and \( R \) be the arbitrary point on that coordinate of the sensing plane. Furthermore, let \( P \) be the intersection between a perpendicular line which pass through \( R \) and the object plane, \( Q \) be the intersection between the perpendicular line and the hemisphere of soft fingertip, and \( V \) be the intersection between that line and last hemisphere. Let \( \Sigma_k \) through \( \Sigma_0 \) be the \( k \)-th through 0-th coordinate systems of the finger, respectively.

An object is fixed so that its orientation angle keeps parallel to the finger coordinate placed under the soft fingertip. We consider an infinitesimal displacement \( Q_kV_k \) as a virtual spring. The displacement of a spring takes place according to an infinitesimal rotation \( \theta_k - \theta_{k-1} \). The key point is that springs \( Q_kV_k \) through \( Q_kP_k \) lie along different direction as illustrated in Fig.4. Then, at \( \Sigma_0 \) frame, the coordinate of \( 0Q_k \) and \( 0R_k \) can geometrically be described as follows:

\[ 0Q_k := \begin{bmatrix} \sqrt{r^2 - (x^2 + y^2)} \sin \theta_k + L + (x - L) \cos \theta_k \\ \sqrt{r^2 - (x^2 + y^2)} \cos \theta_k - (x - L) \sin \theta_k \end{bmatrix}, \]

\[ 0R_k := \begin{bmatrix} L + (x - L) \cos \theta_k \\ - (x - L) \sin \theta_k \end{bmatrix}, \]

where the superscript denotes the coordinate system, and \( (x, y) \) is a coordinate of an arbitrary point \( R \) on the finger. Then, the position vector \( 0Q_k^0R_k \) and the straight line pass through point \( 0Q_k \) and \( 0R_k \) can be expressed as follows:

\[ 0Q_k^0R_k := \begin{bmatrix} \sqrt{r^2 - (x_0^2 + y_0^2)} \sin \theta_0 \\ 0 \\ \sqrt{r^2 - (x_0^2 + y_0^2)} \cos \theta_0 \end{bmatrix}, \]

\[ x - 0R_k \frac{z - 0Q_k}{\sin \theta_0} \cos \theta_0 = t(x, y), \]

where \( 0Q_k \) and \( 0R_k \) stand for \( (x, z) \)-coordinate of \( 0Q_k \) respectively. We derive an intersection \( 0V_k \) between the above straight line and the last fingertip hemisphere.

\[ 0V_k := \begin{bmatrix} t(x, y) \sin \theta_k + L + (x - L) \cos \theta_k \\ t(x, y) \cos \theta_k - (x - L) \sin \theta_k \end{bmatrix}. \]

Using Taylor series expansion of trigonometric function to the first order, we obtain

\[ t(x, y) = -L(\theta_k - \theta_{k-1}) + \sqrt{r^2 - (x^2 + y^2)}. \]

Since \( \theta_k - \theta_{k-1} \) is a constant, parameter \( t(x, y) \) does not depend on the infinitesimal rotation \( \theta_k - \theta_{k-1} \), but on the coordinate \( x \) and \( y \). Therefore, the coordinate and the length of \( V_kQ_k \) can easily be written by

\[ 0V_k^0Q_k := \begin{bmatrix} \left( \sqrt{r^2 - (x_0^2 + y_0^2)} - t(x, y) \right) \sin \theta_k \\ 0 \\ \left( \sqrt{r^2 - (x_0^2 + y_0^2)} - t(x, y) \right) \cos \theta_k \end{bmatrix}, \]

\[ V_kQ_k = \sqrt{r^2 - (x_0^2 + y_0^2)} - t(x, y), \]

where no superscript means a scalar value.

Next, we consider another line segment \( ^kP_k^kQ_k \) shown in Fig.4. Since we formulate the rotational contact model by using a discrete infinitesimal angle \( \theta_k - \theta_{k-1} \), an infinitesimal line segment \( ^kP_k^kQ_k \) can not be ignored. Therefore, when \( k \) takes certain value \( k' \), the line segment \( ^kP_k^kQ_k \) whose length is shorter than that of \( ^kV_k^kQ_k, k = n, n-1, \ldots, k' + 1 \) exists. Also the superscript \( k \) stands for a coordinate system \( \Sigma_k, ^kP_k^kQ_k \).
and the length of \( P_k Q_k \) can respectively be written as follows:

\[
k_k P_k = \begin{bmatrix} x \\ y \end{bmatrix} \left( r - (L + x) \sin \theta_k \right) / \cos \theta_k, \tag{10}\]

\[
k_k Q_k = \begin{bmatrix} x \\ y \end{bmatrix} \sqrt{r^2 - (x^2 + y^2)}, \tag{11}\]

\[
P_k Q_k = \sqrt{r^2 - (x^2 + y^2)} - \left( r - (L + x) \sin \theta_k \right) / \cos \theta_k. \tag{12}\]

where \( dS \) stands for a sectional area of the virtual spring.

Now, let us convert the force equation (16) to a pressure equation by considering the deformation of a virtual spring as incompressible material component shown in Fig.5.

![Fig. 4. Rotational contact model](image)

![Virtual spring](image)

![Virtual spring](image)

We can obtain a relation between the shrunk sectional area \( dS' \) and the pre-deformed area \( dS \) as

\[
dS = \frac{t(x, y)}{\sqrt{r^2 - (x^2 + y^2)}} dS'. \tag{17}\]

Therefore, eq. (16) can be rewritten as follows:

\[
dP = \frac{dF}{dS'} = E \left\{ \sum_{k'=k+1} \frac{1}{\sqrt{r^2 - (x^2 + y^2)}} \left( t(x, y) - \left[ \frac{r - (L + x) \sin \theta_k}{\cos \theta_k} \sqrt{r^2 - (x^2 + y^2)} \right] \right) \right\} \times \frac{t(x, y)}{\sqrt{r^2 - (x^2 + y^2)}} \tag{18}\]

where \( dP \) means a pressure acting on the arbitrary point \( R \) of the finger. Additionally, the second term of eq. (18) can be approximated to zero, when we regard the rotation angle \( \theta_k - \theta_{k-1} \) as a small infinitesimal value. Namely, when the rotation angle increases continuously, the virtual spring \( Q_k P_k \) can be ignored as shown in Fig.4. Thus, eq. (18) can be converted to the following equation:

\[
dP = E \sum_{k'=k+1} \frac{1}{\sqrt{r^2 - (x^2 + y^2)}} \left( t(x, y) - \left[ \frac{r - (L + x) \sin \theta_k}{\cos \theta_k} \sqrt{r^2 - (x^2 + y^2)} \right] \right) \times \frac{t(x, y)}{\sqrt{r^2 - (x^2 + y^2)}} \tag{19}\]

where \( n - k' \) provides the number of virtual springs \( Q_k P_k(Q_k = n, n - 1, \ldots, k' + 1) \) on the arbitrary point \( R \), when the finger rotates from 0 to \( n \) as illustrated in Fig.4. Thus, \( dP \) can be expressed by summing up all pressure values that small line segment \( P_k Q_k \) through \( Q_k V_k \) exert on the point \( R \) by the infinitesimal rotation \( \theta_k - \theta_{k-1} \). Note that we can obtain a new theoretical pressure equation on the sensing plane of the finger.

On the right side of eq. (19), all the terms except Young’s modulus \( E \) correspond to a strain in terms of mechanics of materials.

### C. Formulation of Rolling Contact

Now, let us consider a rolling effect of an object on the top of a soft fingertip. We assume that the object rolls on the soft fingertip retaining a certain displacement of the soft fingertip as shown in Fig.6.
Let $P$ be the intersection between a perpendicular line $QR$ and the flat surface of the pre-rolled object, $Q$ be the intersection between that line and the hemisphere of the fingertip, and $\theta_{obj}$ means an initial angle of the object. After the object rolls by angle $\beta$ around the top of the fingertip retaining a certain deformation, $P$ and $Q$ move to point $P'$ and $Q'$ respectively. Let $dP$ and $dP'$ be the pressure values of the virtual spring $QR$ and $Q'R$ on the arbitrary point $R$.

![Fig. 6. Rolling contact](image)

Then, the equation of the contact plane can be described as

$$
(x - p \sin \theta_{obj}) \sin \theta_{obj} + (z - p \cos \theta_{obj}) \cos \theta_{obj} = 0,
$$

where $p$ corresponds to $r - L \sin \theta_k$ and $\theta_k$ means a gradient angle of the finger. Let $R_k$ be the $x$-coordinate of the point $R$. Additionally, points $P$ and $Q$ and the straight line $PQ$ can be written by

$$
P : \begin{bmatrix} s_{pl} \cdot \sin \theta_{obj} + x \\ y \\ s_{pl} \cdot \cos \theta_{obj} \end{bmatrix},
$$

$$
Q : \begin{bmatrix} s_{sph} \cdot \sin \theta_{obj} + x \\ y \\ s_{sph} \cdot \cos \theta_{obj} \end{bmatrix},
$$

where

$$
s_{pl} = r + (x - L) \sin \theta_k,
$$

$$
2 \sin \theta_{obj} \cos \theta_{obj} = x,
$$

parameters $s_{pl}$ and $s_{sph}$ denote each value associated with the contact plane and the hemisphere respectively. Then, we can obtain a pressure difference at the point $R$ due to the object rolling as follows:

$$
\delta dP = dP' \cdot \cos(\theta_{obj} - \beta) - dP \cdot \cos \theta_{obj}
$$

$$
= E \left\{ \left( 1 - \frac{s_{pl}}{s_{sph}} \right) \frac{s_{pl}}{s_{sph}} \cos(\theta_{obj} - \beta) - \left( 1 - \frac{s_{pl}}{s_{sph}} \right) \frac{s_{pl}}{s_{sph}} \cos \theta_{obj} \right\},
$$

where

$$
s'_{pl} = r - x \sin(\theta_k - \beta) - L \sin \theta_k,
$$

$$
s'_{sph} = -x \sin(\theta_k - \beta) + \sqrt{r^2 - (x^2 + y^2) + x^2 \sin^2(\theta_k - \beta)}.
$$

Parameters $s'_{pl}$ and $s'_{sph}$ denote same parameters after the object rolls as seen in Fig.6. Finally we can derive an integrated pressure equation from eqs. (19) and (27) as follows:

$$
dP_{new} = dP + \delta dP.
$$

Note that the $dP_{new}$ indicates a total pressure value that appears on an arbitrary point $R$ on the top of the sensing plane, after the finger rotates $\theta_k$ and rolls by angle $\beta$.

### III. Simulation results

In this section, we evaluate a pressure center of the pressure distribution and a total force appears on the sensing plane. According to the deformation of the soft fingertip, a certain pattern of pressure distribution appears on the top of the sensing plane. The pressure distribution can be described by using the theoretical pressure equation (30). As shown in Fig.7-(a), first we maintain the finger in condition that the soft fingertip contacts with a fixed object that is in parallel along the finger. We tilt the finger to 7[deg] holding upper wall fixed. Continuously, we fix the finger and make the wall roll by 15[deg] to the arrow direction illustrated in Fig.7-(b). That is, the final angle $\theta_{obj} - \beta$ in the coordinate system on the sensing plane reaches to -8[deg], where $\theta_{obj}$=7[deg] and $\beta$=15[deg].

![Fig. 7. Pattern diagram for simulation](image)

Let $x$ and $y$ be the coordinate of the sensing plane. We configure the finger length as 40[mm], the infinitesimal angle $\theta_k - \theta_{wall}$ as 0.01[deg] and the thickness of the finger as 3.0[mm], which is equivalent to the dimensions of an apparatus that we made for this paper. Fig.8 represents a transition of pressure center on the top of the sensing plane with respect to 3D and 2D views. The $x$-$y$ values represent the number of a tactile sensor cell. This coordinate system has a coordinate of 1 through 10. Then, the center point of the sensing plane is (5.5,5.5).

Fig.8 represents simulation results of the path of the pressure center on the sensing plane. Fig.8-(a) shows the 3D view.
of the path and Fig.8-(b) and (c) represent its top and side views. While the finger is rotating from 0[deg] to 7[deg], the pressure center moves slightly toward positive direction along x-axis. After that, while the object is rolling by 15[deg] with the fixed finger, the pressure center also moves slightly toward negative direction along the same axis. The center does not move along y-axis, and y-coordinate of the center remains 5.5. Fig.8-(d) shows the total force exerted on the sensing plane. When the angle of the finger moves from 0[deg] to 7[deg], the total force increases quadratically as shown in [6]. While the object is rolling on the top of the soft fingertip, the total force varies hardly at all.

![Fig. 8. Simulation result](image)

(c) Side view  
(d) Total force

Fig. 8. Simulation result

### IV. EXPERIMENTAL RESULTS

#### A. Setup

In this paper, we validate our theoretical equation (30) by means of a high performance tactile sensor shown in Fig.9-(a). This sensor consists of 10×10 sensing points, which is referred to as cells. Output of each cell has eight bit resolution, and its maximum amplitude of the pressure for each cell is referred to as cells. Output of each cell has eight bit resolution, and its maximum amplitude of the pressure for each cell is approximately 19.6[N/cm²]. Furthermore, the sensing area can be calculated as \( \sqrt{2} [\text{mm}^2] \) for each, and the thickness of the sensor is less than 0.1[mm]. Its sampling time is less than 1[msec]. We have built a soft fingertip by polyurethane gel, as shown in Fig.9-(b). Its radius is 10[mm], and Young’s modulus \( E \) is estimated at 0.3636[MPa] by tensile test.

We underlay that sensor under the soft fingertip and carry out a contact experiment as shown in Fig.7. As shown in Fig.10, we have fabricated a simple apparatus so that the finger can rotate and roll for a rigid top plate simultaneously. After the finger rotates, it can be fixed with holding rotation angle of the finger. Moreover, that finger can be rotated around a rolling axis. Fig.10-(a) shows a general view of the apparatus, the finger is set in parallel with the top plate, and a soft fingertip is attached on that finger. We put a tactile sensor on a finger surface and place the soft fingertip on the top of that sensor. As well as simulation setting, we make the finger rotate around a rotational axis by 7[deg] and roll around a rolling axis by 15[deg]. We can realize the rolling of the object by rolling the finger against the fixed top plate.

![Fig. 9. Components for experiment](image)

(a) Tactile sensor  
(b) Soft fingertip

Fig. 9. Components for experiment

![Fig. 10. Apparatus for experiment](image)

(a) General view  
(b) Diagonal view

Fig. 10. Apparatus for experiment

#### B. Experimental results

Fig.11 shows a measured path of the pressure center. When the fingertip contacts to the plate at the beginning, the pressure center locates at (7.3,4.6) approximately. The center moves toward (5.9,5.5) as the rotational angle increases. The lower part of Fig.11-(b) and the outset of plotted point in Fig.11-(a) and (c) are caused by the rolling of the finger. In terms of the evaluation of the total force plotted in Fig.11-(d), a desirable tendency can be obtained as shown in Fig.12-(a) and (c) are caused by the rolling of the finger. In terms of the evaluation of the total force plotted in Fig.11-(d), we can find a difference of the total force comparing to the simulation result plotted in Fig.8-(d).

Fig.12 shows a comparison between both results. With respect to the rolling contact, a desirable tendency can be obtained as shown in Fig.12-(a). As the rolling contact proceeds, both pressure centers move along the negative direction of x-axis. However, both transition distances of the pressure center are different each other. Additionally in terms of the rotational contact, also a preferable result can be obtained between simulation results and the last half of experimental
results. That is, the pressure center moves along the same
direction. But almost all parts of the experimental result with
respect to the rotational contact are quite different with the
simulation result. It is attributed to a problem that the tactile
sensor can not measure an accurate value of the pressure in
terms of low measurement range.

![Graphs and tables showing comparison of pressure centers in 3D view, top view, side view, and total force](image)

Fig. 11. Experimental result

Fig.12-(b) shows a top view of the comparison of the
pressure center. Similarly, both paths of the pressure center
can be seen along a line, which satisfy that y-coordinate is 5.5
in rolling contact. The pressure center in the experiment widely
spread in the beginning of the rotational contact. Throughout
this result, we can understand a performance limitation of the
tactile sensor at low measurement range.

V. CONCLUSION

In this paper, we have newly introduced a rotational contact
model between a soft fingertip and an object. By adopting
a virtual spring into the soft fingertip, we have considered a
static force equation, which is finally converted to a new pres-
sure equation. Furthermore, we have combined the pressure
equation of the rotational contact of the finger and that of
rolling contact of the object.

We have obtained a preferable tendency in terms of the
path of the pressure center. However, both locations of the
pressure center between simulation results and experimental
results during the rolling contact are different each other.
It results from the assumption that an elastic force acting
on an infinitesimal virtual spring exerts only on that spring
itself. We do not consider in this paper that the elastic force
exerts on other infinitesimal virtual springs inside the soft
fingertip. Furthermore, experimental results imply that the
pressure center value is not useful at all while the contact
between the object and the soft fingertip is not enough, that
is, the contact is in the beginning of the rotation step of the
finger. It is attributed to a performance itself that individual
differences of the tactile sensor cell are different each other.
The pressure value is not reliable in the initial step of the
rotational contact, because of the performance of the sensor.
If the tactile sensor can be improved, this problem will be able
to solve in future.

![Graphs showing Comparison of results](image)

Fig. 12. Comparison of results

We conclude that it is necessary to improve our rotational
contact model even further so that we can use these pressure
information to grasp and manipulate an object stably by two
fingered hand equipped with soft fingertip.

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