Electromagnetic interpretation of the quarter-wave stack condition by means of the phase calculation in Bragg fibers

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This paper presents an expression for the eigenvalue equation and fields in a Bragg fiber by calculating the phase change that is based on geometrical optics. The quarter-wave stack condition makes it possible to consider that the Bragg fiber has approximately no cladding from the electromagnetic point of view, despite the fact that the Bragg fiber has a periodic cladding. As a result, its eigenvalue equation can be represented in terms of the zeros of Bessel functions and only the core parameters, for a specific case. The eigenvalue equations for HE and EH modes in the Bragg fiber have a formal equivalence to those for TE and TM modes, respectively, in the circular metallic waveguide. Results obtained are in agreement, under a specific limit, with those derived by an asymptotic expansion method. © 2006 Optical Society of America

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1. INTRODUCTION

Photonic crystal fibers have been studied owing to their peculiar properties: the possibility of low loss, low dispersion, and negligible optical nonlinearities. A holey fiber, namely a microstructured fiber (one of photonic crystal fibers), sustains an optical wave through the index guiding and has attained a low loss of 0.28 dB/km at 1550 nm. Other types of photonic crystal fibers are photonic bandgap fibers characterized by light localization originating from the periodicity. Photonic bandgap fibers having a hollow core are divided into two kinds of fibers depending upon the cladding structure. The first kind of fiber has air-hole cladding, and the second kind is a Bragg fiber that has a cylindrical symmetry and a periodic cladding with high and low refractive indices. An air-silica Bragg fiber has been demonstrated recently.

Electromagnetic properties of photonic crystal fibers are usually treated by solving Maxwell’s equations in various ways. Main calculation methods resort to numerical means because of their structural complexity. The microstructured optical fiber has been analyzed by the effective index model, the multipole method, and the FDTD method. The Bragg fiber has been analyzed by the transfer matrix, supercell, and biorthonormal basis methods. Use of the structural symmetry enables us to reduce the calculation required. There are some trials that get good result with moderate manipulations by the introduction of an approximation. Asymptotic expansion methods have been used to analyze the Bragg fiber. The approach by geometrical optics is helpful to offering the physical insight into fiber characteristics. Although step- and parabolic-index optical fibers have widely been analyzed by using Maxwell’s equations, geometrical optics has provided another prospect.

This paper presents the eigenvalue equations and fields in the Bragg fiber using the phase calculation based on a purely geometrical optics approach. These equations and fields are obtained under the quarter-wave stack (QWS) condition and are represented in a simple form consisting of only the core parameters. The results agree with those obtained by applying an asymptotic expansion method to wave functions derived from Maxwell’s equations. The present approach is also useful for understanding a similarity in characteristics between the Bragg fiber and the circular metallic waveguide.

2. DESCRIPTION OF RAY

A Bragg fiber has a cylindrical symmetry and has a hollow core surrounded by the periodic cladding with alternately high and low indices. Although the cladding is assumed to extend infinitely, only the two cladding bilayers are depicted in Fig. 1 for simplicity. The core radius is , and its refractive index is . One of cladding bilayers has thickness and index , and the other has thickness and index . The period of the cladding is denoted by . It is assumed throughout this paper that the core radius is relatively large so as to meet with the asymptotic expansion approximation. A cylindrical coordinate system is used, with being the propagation direction of light.

We must determine a ray trajectory to evaluate the accumulated phase changes. Successive rays reflected by an identical angle at the core-cladding boundary generates an envelope, namely a concentric circle with radius . The concentric circle is called the caustic, where a ray experiences a phase change of . A ray starting from the caustic reaches the core-cladding boundary and reflects or refracts there. The ray penetrating into the cladding is sent back to the core after being reflected at the cladding layer interfaces. The ray arrives at the caustic again. Let us consider the ray propagating in the core at first.
The straight line PQ touches the caustic at its middle point S (see Fig. 2). We put $\phi_{r,ca}$, $\phi_{\theta,ca}$, and $\phi_{z,ca}$ as the ray angles with respect to the $r$, $\theta$, and $z$ axes, respectively, at the point S. Since the ray is tangent to the caustic, we get $\phi_{r,ca} = \pi/2$. One obtains $\phi_{z,ca} = \phi_{z,c}$, because the ray angle to the $z$ axis is kept unchanged in the core. Then we have

$$\cos \phi_{z,ca} = \sin \phi_{z,c}$$

(4)

from the above result and an equality of the direction cosine, $\cos^2 \phi_{r,ca} + \cos^2 \phi_{\theta,ca} + \cos^2 \phi_{z,ca} = 1$.

In $\triangle O'SQ$ one obtains $O'Q^2 = r_{ca}^2 + (PQ/2)^2$, because $\phi_{r,ca} = \pi/2$. In $\triangle O'QSQ$ we have $O'Q^2 = r_{ca}^2 + (PQ/2) \times \cos \phi_{z,c}^2$. Putting these two relations equal, we get an expression about the caustic radius $r_{ca}$:

$$r_{ca} = r_c \left[ 1 - \left( \frac{\cos \phi_{z,c}}{\sin \phi_{z,c}} \right)^2 \right]^{1/2},$$

(5)

using Eq. (1). The above expression gives the caustic radius as a function of the ray angles and the core radius $r_c$.

For later use, expressions including only the ray angles will be related to those containing the caustic radius. Projection of $PQ/2$ onto a plane perpendicular to the $z$ axis is equal to $(r_c - r_{ca})^{1/2}$. This yields an equality

$$OQ = r_c \left[ 1 - \left( \frac{\cos \phi_{z,c}}{\sin \phi_{z,c}} \right)^2 \right]^{1/2}.$$
\[
\frac{r_c \cos \phi_c}{\sin \phi_{zc}} = (r_c^2 - r_{ca}^2)^{1/2}. \tag{6}
\]

In addition, another expression for the rational displacement, \(\delta \theta = 2 \cos^{-1} (r_{ca}/r_c)\), results in
\[
\cos^{-1} \left( \frac{\cos \phi_a}{\sin \phi_{zc}} \right) = \frac{1}{2} \sin^{-1} \left( \frac{2 \cos \phi_c \cos \phi_{zc}}{\sin^2 \phi_{zc}} \right) = \cos^{-1} \left( \frac{r_{ca}}{r_c} \right)
\]
using a relationship\(^{17}\)
\[
\sin^{-1} \left( \frac{2 \cos \phi_c \cos \phi_{zc}}{\sin^2 \phi_{zc}} \right) = 2 \cos^{-1} \left( \frac{\cos \phi_{zc}}{\sin \phi_{zc}} \right). \tag{7}
\]

B. Ray in the Cladding

Let us consider a ray travelling from Q to T in the cladding layer “a,” as shown in Fig. 3. In \(\triangle OQT\) one obtains \(QT = [(r_c + a)^2 + (QT \cos \phi_{ca})^2]^{1/2}\) using \(TTQ = QT \cos \phi_{ca}\). Application of the second cosine theorem to the triangle produces a second-order equation of \(QT\):
\[
\sin^2 \phi_{za} \frac{QT^2}{2} + 2r_c \cos \phi_{za} \frac{QT}{2} - 2r_c a = 0. \tag{9}
\]

If the cladding thickness \(a\) is sufficiently smaller than the core radius \(r_c\), then \(QT\) is approximated by
\[
\frac{\sqrt{QT}}{\cos \phi_{za}} = \frac{a}{\cos \phi_{za}}. \tag{10}
\]

Equation (10) shows that \(\angle TVQ = \angle R\), namely \(\phi_{za} = \pi/2\). The translation between points Q and T along the \(z\) direction is given by
\[
\delta z_a = \frac{\sqrt{QT} \cos \phi_{za}}{\cos \phi_{za}} = \frac{a \cos \phi_{za}}{\cos \phi_{za}}. \tag{11}
\]

An equality, \(\frac{\sqrt{QT} \cos \phi_{za}}{r_c \cos \phi_{za}} = \frac{a \sin \delta \theta_a}{r_c \cos \phi_{za}}\), geometrically obtained leads to a rational displacement in the cladding layer “a” as
\[
\delta \theta_a = \sin^{-1} \left( \frac{a \cos \phi_{za}}{r_c \cos \phi_{za}} \right) = \frac{a \cos \phi_{za}}{r_c \cos \phi_{za}}. \tag{12}
\]

This means that the angle displacement of the ray in the cladding layer is smaller by the order of \(a/r_c\) than that in the core. In addition, \(\delta \theta_a\) is greatly close to zero because of a property of \(\phi_{za} = \pi/2\).

For the cladding layer “b” we obtain results in which parameter “a” is replaced with “b” in Eqs. (10)-(12).

C. Relations between Ray Angles and Modal Parameters

The propagation constant \(\beta\) and the lateral propagation constant \(\kappa\) are indicated by
\[
\beta = n_c k_0 \cos \phi_{zi} \tag{13}
\]
and
\[
\kappa = [(n_c k_0^2 - \beta^2)^{1/2}] \tag{14}
\]
for \(i = a, b\) and c. Here, \(k_0\) denotes the vacuum wavenumber. By using above relations, parameters including the ray angle can be rewritten as follows:

Fig. 3. View of ray propagating from point Q on the core-cladding boundary to point T on the outer side of cladding layer “a” with thickness \(a\) and index \(n_c\). The ray makes angles \(\phi_{za}\) and \(\phi_{za}\) with respect to the \(r\), \(\theta\), and \(z\) axes, respectively, at the inner interface of each layer. The \(T_Q\) is the projection of the point \(T\) onto a plane perpendicular to the \(z\) axis and including the point \(Q\). Translations of \(QT\) along the \(\theta\) and \(z\) directions are indicated by \(\delta \theta_a\) and \(\delta z_a\). The core radius is denoted by \(r_c\).

\[
n_c k_0 \sin \phi_{zc} = n_c k_0 (1 - \cos^2 \phi_{za})^{1/2} = \kappa. \tag{15}
\]

The angle dependence of fields is represented by \(\exp(-i \nu \phi)\) with \(\nu\) being the azimuthal mode number. The phase change caused by a one-turn trip along the periphery of the core is estimated to be \((n_c k_0 \cos \phi_{za}) \pi r_c\). This corresponds to a phase change of \(2 \pi \nu\). Therefore this equality results in the azimuthal mode number
\[
\nu = n_c k_0 r_c \cos \phi_{za} \tag{16}
\]

Use of the ray angle at the caustic gives another expression for \(\nu\)
\[
\nu = n_c k_0 r_c \cos \phi_{za} = \kappa r_c \tag{17}
\]
with the aid of a relation, \((n_c k_0 \cos \phi_{za}) \pi r_c = 2 \pi \nu\). It is found from Eq. (17) that the azimuthal mode number \(\nu\) is proportional to the caustic radius \(r_c\) for a fixed value of \(\kappa\). This implies that \(\nu = 0\) is equivalent to \(r_c = 0\). In addition, we get
\[
n_c k_0 r_c \cos \phi_{zc} = [(\kappa r_c)^2 - \nu^2]^{1/2} \tag{18}
\]
by using the equality of the direction cosine.

The caustic radius is related to propagation parameters as
\[
r_c = \frac{\nu}{\kappa} \tag{19}
\]
by substituting Eqs. (15) and (18) into Eq. (5). We see from Eq. (19) that the caustic radius \(r_c\) becomes zero for \(\nu = 0\). This indicates that both TE and TM modes with \(\nu = 0\) are the meridional rays traversing the core center.
3. DERIVATION OF EIGENVALUE EQUATIONS BY MEANS OF THE PHASE CALCULATION

Eigenvalue equations of the Bragg fiber will be derived under the QWS condition in this section. Before doing so, the phase change in the cladding will be considered.

A. Phase Change in the Cladding

One can approximate the ray angle along the \( \theta \) direction in the cladding layers “a” and “b” by

\[
\phi_i = \pi/2 \quad (i = a, b),
\]

as described before. Equation (20) suggests that the phase change in the cladding is equivalent to that along the \( r \) direction. In fact, a quantity relating to the ray angle along the \( r \) direction is expressed by

\[
n_k r \cos \phi_i \equiv \kappa_i \quad (i = a, b)
\]

by utilizing Eqs. (13), (14), and (20), and the property of the direction cosine.

The phase change due to a ray travel in the cladding layer “a” becomes

\[
P_a = n_k r \cos \phi_a = n_k r \cos \phi_{za} = n_k r \cos \phi_{za} = n_k r \cos \phi_{za} + \kappa \sin \phi_{za} = \kappa \cos \phi_{za} \tag{21}
\]

with the help of Eqs. (10)–(12) and (21). Similarly, one obtains a phase change of \( P_b = n_k r \cos \phi_{cb} \) for the cladding layer “b.” The phase changes in the cladding layers “a” and “b” are approximately equal to those in the \( r \) direction, because the cladding thickness is assumed to be sufficiently small compared with the core radius. Equation (22) indicates that the phase change due to one-way ray travel through each cladding layer is denoted by the product of the lateral propagation constant and the layer thickness.

For the QWS condition, that is, \( \kappa_a = \kappa_b = \pi/2 \), the round-trip path through \( m \) bilayers produces a phase change of \( 2\pi m \). When an outward ray is reflected at the core-cladding boundary or at the interface between the outer side of cladding layer “b” and the inner side of layer “a,” the phase change caused by its reflection shows an identical value. Hence, the phase at the core-cladding boundary is equivalently the same as that at \( r = r_c + m \Lambda \), with \( m \) being a natural number.

When an outward ray is reflected at the interface between the outer side of cladding layer “a” and the inner side of layer “b,” the phase change of this ray is different from the former case by an additional phase of \( \pi \), according to Fresnel’s formula. In this case, the total phase change from the core-cladding boundary to the interface, \( r = r_c + m \Lambda + a \), is a multiple of \( 2\pi \), including the phase change due to its round-trip path.

In summary, if the QWS condition is satisfied in the cladding, rays reflected back from any interface in the cladding contribute in phase to the phase change at the core-cladding boundary. Accordingly, when discussing field characteristics in the core, we consider only the core-cladding boundary and can neglect the contribution of the cladding. This supports that the Bragg fiber exhibits a property similar to that of the circular metallic waveguide

waveguide, which has a hollow core and a perfectly conducting wall. In the original work of the Bragg fiber, a cladding condition \( \kappa_a + \kappa_b = \pi \) was determined by a full variation between two adjacent zeros of Bessel functions. However, the meaning of the QWS condition is explained here by the phase change.

B. Eigenvalue Equation Derived by the Phase Calculation

When the QWS condition is satisfied in the cladding, an eigenvalue equation of the Bragg fiber can be derived by calculating the accumulated phase changes in the core. A phase change during a one-period path from one to another caustics is divided into several effects, such as the direct translation \( F_\Omega \), translation \( \delta \zeta_c \) along the \( z \) direction, rational displacement \( \delta \theta_c \), and an additional phase change \( \Phi_{add} \). The total phase change must be a multiple of \( 2\pi \) so that standing waves may be constituted in any cross section of the core. It follows that

\[
n_k \sqrt{F_\Omega} - n_k \cos \phi_{zc} \delta \zeta_c - n_k \cos \phi_{dc} r_c \delta \theta_c + \Phi_{add} = 2\pi \mu,
\]

with \( \mu \) being an integer and corresponding to the radial mode number. The additional phase change \( \Phi_{add} \) consists of the phase change \( \Phi_c \) because it touches the caustic, and a phase change \( \Phi_R \) due to reflection at the core-cladding boundary:

\[
\Phi_{add} = \Phi_c + \Phi_R, \tag{24}
\]

where \( \Phi_c = -\pi/2 \) and \( \Phi_R \) is determined from Fresnel’s formula:

\[
\Phi_R = \begin{cases} 
\pi & \text{the S component in all cases and the } \\
0 & \text{the P component with large incident angle } \\
\end{cases}
\]

Substitution of Eqs. (1)–(3) and (7) into Eq. (23) leads to an eigenvalue equation

\[
2n_k r_c \left[ \cos \phi_{vc} - \cos \phi_{dc} \cos^{-1} \left( \frac{\cos \phi_{vc}}{\sin \phi_{zc}} \right) \right] + \Phi_c + \Phi_R = 2\pi \mu
\]

that is represented by the ray angles. Use of Eqs. (15), (16), and (18) produces another form of the eigenvalue equation, including the modal parameters \( \kappa_c \) and \( \nu_c \):

\[
2 \left[ \left( \kappa_c r_c \right)^2 - r_c^2 \right]^{1/2} - r_c \cos^{-1} \left( \frac{\nu_c}{\kappa_c r_c} \right) \right] + \Phi_c + \Phi_R = 2\pi \mu.
\]

Inserting Eqs. (6) and (7) into Eq. (26) and using Eq. (15), we can write the eigenvalue equation as
The eigenvalue equations obtained in Eqs. (31) and (32) are very simple and include only the core parameters. They are in accordance with those for TM and TE modes, respectively, in the circular metallic waveguide. This fact can be explained as follows. When the cladding layer thickness is sufficiently small compared with the core radius and the cladding parameters exactly satisfy the QWS condition, any cladding interfaces contribute in phase to the phase change at the core-cladding boundary, as described before. Then, it is possible to regard the Bragg fiber as one having no cladding from the electromagnetic point of view.

4. CORE FIELDS DERIVED BY THE PHASE CALCULATION

When all field components have a temporal factor of \( \exp(\text{i} \omega t) \), the longitudinal \( E_z \) and \( H_z \) components in the Bragg fiber satisfy the Helmholtz equation:

\[
(\Delta + k^2)\psi = 0,
\]

where \( \psi \) stands for \( E_z \) or \( H_z \) and \( k \) denotes the wavenumber in a medium. Therefore it is possible to express the magnitude of field in terms of the phase change. The field at a point \((r, \theta, z)\) can be calculated by

\[
\psi(r, \theta, z) = \sum_j A_j(r, \theta, z) \exp[-\text{i}kS_j(r, \theta, z)]
\]

in the limit of an infinitesimal wavelength. Here, \( A_j \) indicates the amplitude and \( S_j \) indicates the eikononal. Accordingly, the product \( P_j = kS_j \) stands for the phase. Summation must be performed over all rays gathering to the point \((r, \theta, z)\).

A. Phase Change

Let us evaluate the core field at a point \( D(r, \theta, z) \) locating between the caustic and the core-cladding boundary, as shown in Fig. 4. One ray leaves a point \( A \) on the caustic surface, travels in the direction tangent to the caustic and arrives at a reflection point \( C \) on the core-cladding boundary. The ray propagates toward the point \( D \) after reflection. The other ray tangent to the caustic leaves another point \( B \) and directly reaches the same point \( D \). Two rays constitute the core field at the point \( D \).

The angles of the reflected ray, \( CD \), at the core-cladding boundary are the same as those at the point \( P \) in Fig. 2. The angle between \( BD \) and the \( z \) axis is equal to \( \phi_{zc} \). The distance from the point \( B \) to point \( D \) for the direct ray is

\[
BD = \frac{(r_c^2 - r_{ca}^2)^{1/2}}{\sin \phi_{zc}}.
\]

The distance from the point \( A \) to point \( D \) via point \( C \) for the reflected ray is written by

\[
\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{PQ} - \overrightarrow{BD} = \left[2(r_c^2 - r_{ca}^2)^{1/2} - (r^2 - r_{ca}^2)^{1/2}\right] \frac{1}{\sin \phi_{zc}}
\]

with the aid of Eqs. (1), (35), and (6). The translation of the direct ray, \( BD \), along the \( z \) direction becomes
situated at the cross point of the line point C on the core-cladding boundary. Points HA, HB, and H0 are other point A and arrives at the same point D after reflecting at a
gent to the caustic surface leaves a point B on the caustic and
caustic to point D

\[ \frac{z}{r} = \frac{r_{ca}}{r_{c}} \]  \hspace{1cm} (41)

for the direct ray and

\[ \frac{z}{r} = \left( \frac{AC + CD}{n_{ca} \cos \phi_{ca}} \right) \]  \hspace{1cm} (42)

for the reflected ray. It should be noted that the phase change is measured from the standard point H0.

The phase change of the direct ray at point D can be represented as follows:

\[ P_{d}(r, \theta, z) = \left( n_{ca} k_{0} BD + n_{ca} k_{0} \cos \phi_{ca} \right) \text{arc BH}_{D} \]

\[ = \beta z + \sqrt{r_{ca}^2 - r_{c}^2} \]  \hspace{1cm} (43)

which is obtained by inserting Eqs. (35), (39), and (41) into the first line of Eq. (43) and then by utilizing Eqs. (13), (15), and (17). Note that we must use the ray angles at the caustic in this case. The phase change of the reflected ray caused by travel from the standard point H0 to D is obtainable as

\[ P_{f}(r, \theta, z) = \left( n_{ca} k_{0} \frac{AC + CD}{n_{ca} \cos \phi_{ca}} \right) \text{arc HA}_{A} \]

\[ + \left( n_{ca} k_{0} \cos \phi_{ca} \right) \frac{z}{r} \]  \hspace{1cm} (44)

with the help of Eqs. (36), (40), (42), (13), (15), and (17). Employment of the eigenvalue Eq. (28) in Eq. (44) provides a new expression for the phase change \( P_{f} \):

\[ P_{f}(r, \theta, z) = \beta z + \sqrt{r_{ca}^2 - r_{c}^2} \]  \hspace{1cm} (45)

\[ - \kappa_{i} (r_{ca}^2 - r_{c}^2)^{1/2} + 2 \pi \mu - \Phi_{0} - \Phi_{R}. \]

**B. Amplitude**

The amplitude of each ray in this case can be calculated by\(^{18}\)

\[ A(r, \theta, z) = \frac{A_{0}}{s^{1/2}}. \]  \hspace{1cm} (46)

Here, \( A_{0} \) indicates a constant and \( s \) is the phase change caused by the path leaving from the caustic except for the constant term. It is measured from the caustic in this case.

In calculating the amplitude of the direct ray (see Fig. 4), \( s \) in Eq. (46) corresponds to \( n_{ca} k_{0} BD - n_{ca} k_{0} \cos \phi_{ca} \times BD \cos \phi_{ca} \), leading to the last term of Eq. (43). Accordingly, we can write down its amplitude as
Use of the last term of Eq. (44) yields the amplitude
\[ A_1(r, \theta, z) = \frac{A_0}{\sqrt{\kappa_c(r^2 - r_{ca}^2)^{1/4}}}. \]  

In the case of the reflected ray, we can get an expression in which the BD in the direct ray is replaced by AC + CD. Use of the last term of Eq. (44) yields the amplitude
\[ A_2(r, \theta, z) = \exp(i\Phi_0)\frac{A_0}{\sqrt{\kappa_c(r^2 - r_{ca}^2)^{1/4}}}. \]  
The exponential term in Eq. (48) reflects the phase change due to reflection at the core-cladding boundary.

C. Core Fields

Insertion of Eqs. (43), (45), (47), and (48) into Eq. (34) produces the core field at point D as
\[ \psi(r, \theta, z) = \frac{2A_0}{\sqrt{\kappa_c(r^2 - r_{ca}^2)^{1/4}}} \times \cos \left[ \frac{\kappa_c(r^2 - r_{ca}^{1/2}) - \nu \cos^{-1} \left( \frac{r_{ca}}{r} - \pi \mu + \frac{\Phi_{ca}}{2} \right)}{r} \right] \times \exp[-i(\beta z + \nu \theta)]\exp \left[ -i \left( \frac{\pi \mu - \Phi_{ca}}{2} \right) \right]. \]  

In Eq. (49) the term \( \exp[-i(\beta z + \nu \theta)] \) represents the field variation in the \( z \) and \( \theta \) coordinates. Terms except for the two exponential factors express the core field for the Bragg fiber as a function of the radial coordinate \( r \) situated between the caustic and the core-cladding boundary. Equation (49) applies for the longitudinal \( E_z \) or \( H_z \) component that satisfies Eq. (33).

D. Core Fields for a Specific Case

For \( \nu/\kappa_c r_{ca} \ll 1 \), the caustic radius \( r_{ca} \) approaches zero, hence \( \cos^{-1}(r_{ca}/r) \) tends to \( \pi/2 \) in Eq. (49). Then the \( r \) dependence of the core field is rewritten as
\[ \psi(r, \theta, z) \propto (-1)^{\nu} \frac{1}{\sqrt{\kappa_c r}} \cos \left[ \kappa_c r - \frac{(2\nu + 1)\pi}{4} \right]. \]  

from Eq. (49). The cosine term takes a different form according to the value of \( \nu \); it is represented by \((-1)^{(\nu+3)/2}\sin(\kappa_c r - \pi/4)\) for odd \( \nu \) and by \((-1)^{\nu}\cos(\kappa_c r - \pi/4)\) for even \( \nu \). Expression (50) is valid for the limit of the infinitesimal wavelength. This limiting case is equivalent to the asymptotic expansion approximation in which \( \kappa_c r \gg 1 \) is assumed.

In the core field of the Bragg fiber, the longitudinal \( E_z \) or \( H_z \) component is represented by \( J_1(\kappa_c r) \exp[i(\nu \theta - \beta z)] \). When its argument \( \kappa_c r \) approaches infinity, the core field can be approximated by
\[ J_1(\kappa_c r) \approx \left( \frac{2}{\pi \kappa_c r} \right)^{1/2} \cos \left[ \kappa_c r - \frac{(2\nu + 1)\pi}{4} \right]. \]  

A comparison of Eq. (50) with Eq. (51) reveals that the core field calculated by the phase change agrees with that evaluated by the asymptotic expansion for \( \nu/\kappa_c r_{ca} \ll 1 \).

Numerical examples on electromagnetic fields have been calculated for hybrid modes under the QWS condition.\(^{21}\) In the case of the EH\(_{11}\) mode, the \( E_z \) component is approximately represented by a sinusoidal function in the core and becomes zero at the core-cladding boundary. The \( H_z \) component for HE\(_{11}\) and HE\(_{22}\) modes has a relative maximum and relative minimum, respectively, at the core-cladding boundary. Conditions \( \nu = 0 \) and \( \partial \phi/\partial r = 0 \) for the exact core field at \( r = r_c \) produce the same expressions as those in the right-hand side of Eqs. (31) and (32), respectively. Application of the same conditions to Eq. (50) at \( r = r_c \) leads to expressions identical to those in the middle term of Eqs. (31) and (32), respectively, provided that a slow change in \( r \) inside the root term is neglected. Thus behaviors in the Bragg fiber are semiquantitatively confirmed by Eq. (50) except for the vicinity of \( r = 0 \). Discrepancy between the previous data and present results can be explained by the ratio \( \nu/(\kappa_c r_{ca}) \). Actually, values of \( \nu/(\kappa_c r_{ca}) = 0.42, 0.25, \) and 0.28, respectively, for HE\(_{11}\), EH\(_{11}\), and HE\(_{22}\) modes.

5. CONCLUSIONS

The eigenvalue equations and core fields for hybrid modes in the Bragg fiber were derived by calculating the phase change, which is based on geometrical optics. The QWS condition allows us to consider that the Bragg fiber has approximately no cladding from the electromagnetic point of view, despite the fact that the Bragg fiber has a periodic cladding. This fact, under a specific case, yields the simple eigenvalue equation [Eq. (31)] for the EH mode and Eq. (32) for the HE mode. These equations are represented in terms of the zeros of Bessel functions and only the core parameters. The former and latter eigenvalue equations have a formal agreement with those for the TM and TE modes, respectively, in the circular metallic waveguide.

The eigenvalue equations and core fields evaluated by the phase calculation agree with those derived by application of the asymptotic expansion to Maxwell's equations under the QWS condition. For a specific case, \( \nu/\kappa_c r_{ca} \ll 1 \), the caustic radius tends to zero, implying that hybrid modes in the Bragg fiber exhibit a property of the TE or TM mode under the QWS condition. The validity of the present theory was confirmed by comparing results with numerical examples. Thus the theory based on the phase calculation gives an electromagnetic insight into the property of the Bragg fiber.

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