Equivalence between in-phase and antiresonant reflection conditions in Bragg fiber and its application to antiresonant reflecting optical waveguide-type fibers

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1. INTRODUCTION

Various types of photonic crystal fibers (PCFs) have been exploited. The guiding principle of PCFs is classified into the total internal reflection and the photonic bandgap (PBG). The microstructured PCF based on the total internal reflection has attained a low loss [1] comparable to that of conventional silica optical fibers. Other types of PCFs are under development.

The Bragg fiber, one of the PCFs, consists of a hollow core and periodic cladding with alternately high- and low-refractive indices [2]. The Bragg fiber confines its fields to the core owing to the Bragg diffraction originating from the periodicity in the cladding. Studies on the Bragg fiber have been revived since its guiding principle is interpreted as one of PBG fibers (PBFs) [3].

The Bragg fiber’s eigenvalue equations are usually difficult to solve. Although an asymptotic expansion approximation is one way to solve them readily [4,5], this method still requires solving a transcendental equation. Eigenvalue equations have been simplified for restricted cases using a cavity model [6] or a quarter-wave stack (QWS) condition, which was originally used for mathematical reasons [2]. Simplified eigenvalue equations have been derived from the viewpoint of wave [5] and geometrical optics [7]. The QWS condition is useful for understanding properties of the Bragg fiber, such as the confinement loss (CL) [8]. Various characteristics, including the CL, have been investigated for the Bragg fiber [2]. The Bragg fiber has been fabricated and its bend loss has been measured [10].

Much attention has been attracted to all-solid PBFs, in which high-index cylinders are embedded in a low-index background. The optical confinement to a low-index core surrounded by a high-index region is understood by several models. One is the antiresonant reflecting optical waveguide (ARROW) model, which was originally analyzed for a high-index single cladding in one dimension [11]. The guiding principle is based on the antiresonant reflection whose phase condition is opposite the characteristics of a Fabry–Pérot resonator. Analysis of the ARROW waveguide has been generalized to a leaky waveguide that has the cladding of multiple antiresonant layers [12]. The antiresonant and resonant conditions are shown there in terms of the accumulated phase at each round trip path for the one-dimensional (1D) and the cylindrically symmetric case. The idea of the ARROW model has been extended to all-solid PBFs in two dimensions [13,14].

ARROW-type fibers have been fabricated as an all-solid PBF [15], and then as a low-index contrast PBF [16]. All-solid PBFs show unique properties of bend loss [17,18] in that the odd- and even-numbered PBGs exhibit greatly different bend losses. The resonant/antiresonant features of cylinders have been used to explain the formation of PBG [19] and a hollow-core square lattice PCF [20]. The PBGs of an all-solid PBF have been observed as predicted by the ARROW model [17].

Properties of all-solid PBFs with two-dimensional (2D) structures have also been investigated by models other than the ARROW model. A scattering model from a single cylinder [21] has been extended to a case where the cylinders are widely spaced apart [22]. In addition, a semianalytical model has been presented [23] in which a transformation of unit cell and a weak guidance approximation are used.

Bragg and ARROW-type fibers have been evolved independently. Recently, the PBG structure has been discussed using a stratified planar ARROW (SPARROW) model [24] in which the generalized ARROW model [12] is applied to the Bragg fiber. A relationship between the PBG and antiresonance has been investigated for these fibers.

The purpose of present paper is to show the relationship of guiding principle between the in-phase condition at the core-cladding boundary and the antiresonant reflection condition
in the cladding consisting of multiple layers by calculating the phase change. The phase is useful for the semiquantitative understanding of physical properties in a simple calculation. A generalized QWS condition is derived from the physical point of view in the Bragg fiber, and it is related to the ARROW model.

This paper is organized as follows. Section 2 transforms the in-phase condition at the core–cladding boundary into the antiresonant reflection conditions at each layer round trip in the Bragg fiber, and obtains a generalized QWS condition. Section 3 discusses eigenvalue equations of the Bragg fiber under the generalized QWS condition. In Section 4, the antiresonant reflection condition is transformed into the in-phase condition in a structure with each satisfying the antiresonant reflection condition. Section 5 shows the usefulness of the generalized QWS condition by numerical examples. Section 6 applies the phase calculation to understanding the PBG dependence of confinement and bend losses.

2. TRANSFORMATION OF IN-PHASE INTO ANTIRESONANT REFLECTION CONDITIONS IN BRAGG FIBER

A schematic of the Bragg fiber is shown in Fig. 1. The Bragg fiber consists of a core, specified by index \( n_a \) and radius \( r_c \), and the periodic cladding with alternately repeating high- (\( n_b \)) and low-refractive indices (\( n_c \)); \( n_a > n_b > n_c \). The period of cladding is \( \Lambda = a + b \), with cladding thicknesses \( a \) and \( b \) corresponding to layers \( a \) and \( b \), respectively. For the CL evaluation, the number \( N \) of cladding pairs is finite. We use a cylindrical coordinate system \((r, \theta, z)\) with \( z \) being the propagation direction of light.

Transverse components of a wave along the radial direction propagate from the core to the cladding, and are then sent back to the core after being reflected at the core–cladding boundary and cladding layer interfaces. Fields constituted can be explained by the phase condition. The phase change is accumulated by the radial translation and an additional phase change \( \Phi_{\text{add}} \) [2]. To determine the phase change due to radial translation, we need the lateral propagation constants denoted by

\[
\kappa_i \equiv [(n_i k_0)^2 - \beta^2]^{1/2} \quad (i = a, b, c),
\]

where \( \beta \) is the propagation constant, \( k_0 = 2\pi/\lambda_0 \) is the wavenumber of the vacuum, and \( \lambda_0 \) is the wavelength of the vacuum.

The additional phase change \( \Phi_{\text{add}} \) consists of two factors: \( \Phi_a = -\pi/2 \), caused by touching the caustics within the core, and \( \Phi_b \) due to the reflection. The \( \Phi_{\text{add}} \) is not needed when considering the phase change in the cladding. The phase change \( \Phi_R \) due to reflection depends on the relative magnitude in refractive indices of layers situated at both sides of the interface. Let the initial position of the phase change be the core–cladding boundary just after reflection within the core. Then, we can write

\[
\Phi_R = \begin{cases} 
0 & \text{for transmission from low- to high-index materials} \\
\pi & \text{for transmission from high- to low-index materials} 
\end{cases}
\]

If all the reflected waves stated above are in phase at the core–cladding boundary, then they are mutually enhanced as a consequence of the constructive interference. This defines the in-phase condition. At first, let us consider the phase condition of the first period alone [see Fig. 2(a)]. Waves reflected from the outer interfaces of cladding layers \( a \) and \( b \) must satisfy the phase change \( \Phi_1 \) \( (i = a, b) \) as

\[
\Phi_a = 2\kappa_a a + \pi = 2\pi q_1, \quad \Phi_b = 2(\kappa_a a + \kappa_b b) = 2\pi q_2 \\
(q_1, q_2: \text{ integers}, q_2 \geq q_1)
\]

simultaneously. The phase change \( \pi \) in the first equation of Eq. (3) accounts for \( \Phi_R \) accompanied by the reflection. It follows from Eq. (3) that

\[
\kappa_a a = \pi q_1 - \pi/2, \quad \kappa_b b = \pi (q_2 - q_1) + \pi/2.
\]

The in-phase condition of waves reflected from the outer interfaces of the \( m \)th cladding layers \( a \) and \( b \) is offered in the first equation of Eq. (b) accompanied by the reflection. It follows from Eq. (A.4) that the accumulated phase changes can be represented as a function of \( q_1, q_2, \) and \( m \) in the periodic structure, as shown in Eq. (A6). Hence, we utilize Eqs. (3) and (4), which takes only the first period into consideration, hereafter.

For \( q_1 = q_2 = 1 \) in Eq. (4), as a special case, one obtains

\[
\kappa_a a = \kappa_b b = \pi/2.
\]

Fig. 1. (Color online) Schematic of Bragg fiber. \( r_c \), core radius; \( n_a \), refractive index of core; \( n_a \) and \( n_c \), indices of layers with thickness \( a \) and \( b \), respectively; \( \Lambda = a + b \), period in cladding; \( n_{\text{ex}} \), external layer index; \( N \), number of cladding pairs; \( n_a > n_b > n_c \).

Fig. 2. (Color online) In-phase and antiresonant reflection conditions in Bragg fiber. (a) In-phase condition at the core–cladding boundary. The open circle indicates a position at which the phase change starts to be measured. (b) Antiresonant reflection condition at each round trip.
The expression in Eq. (5) is the QWS condition, which was initially considered as a (physically meaningful) special case of outgoing flux minimization [2]. Expressions in Eq. (4) are similarly derived from physical requirements of light confinement (minimization of outgoing flux) and are called the generalized QWS condition because the $\pi/2$ (of generalized integer order) corresponds to a quarter-wave in these equations. The generalized QWS condition extends this principle to all antiresonance orders of the cladding layers. The generalized QWS condition is formally equivalent to the so-called central gap point ($P_0$) [24], as shown in Section 3.

A combination of the two equations in Eq. (4) produces the expressions in Eq. (3). Namely, the in-phase condition at the core–cladding boundary is equivalent to the generalized QWS condition in the Bragg waveguide. Under these conditions, waves reflected from each interface are efficiently confined to the core.

For a case where the in-phase condition is satisfied, we consider the phase change $\Phi_i^1 (i = a, b)$ accumulated by the round trip inside each cladding layer $i$ next [see Fig. 2(b)]. For cladding layers $a$ and $b$, we can express the total phase changes as

$$\Phi_a^1 = 2 \kappa_a, \Phi_b^1 = 2 \kappa_b,$$  

with the help of Eq. (4). Note that the phase change due to the reflection in both interfaces is $2 \pi$ for layer $a$ and 0 for layer $b$.

Equation (6) indicates that the phase change at each round trip is an odd multiple of $\pi$. The present result is formally the same as that of the antiresonant reflection condition [12]. However, the present result is different from previous ones as follows: Eq. (6) is derived from the in-phase condition for the periodic cladding system, whereas [12,14] treat neither the periodic system nor the in-phase condition. In the SPARROW model [24] in which a Bragg stack is decomposed into isolated high- and low-index slabs, the behavior of the Bragg stack is approximated by the properties of the individual slabs.

It is summarized that if the in-phase condition at the core–cladding boundary is satisfied in the 1D Bragg waveguide, then we always obtain the antiresonant reflection condition. It has been derived for the Bragg fiber that the phase change in the cladding can be approximated by that along the radial direction in the framework of geometrical optics [7], indicating that this approximation is valid for the limit of an infinitesimal wavelength, namely, for an infinite $V$ parameter [12]. This statement assures that the 1D result also holds for the cylindrically symmetric 2D structure, such as the Bragg fiber.

When Eq. (4) holds, one can observe a standing wave pattern of $(q_1 - 1/2)/2$ cycles and $(q_2 - q_1 + 1)/2$ cycles inside the cladding layers $a$ and $b$, respectively, under an asymptotic expansion approximation (see Appendix B). Actually, for $q_1 = q_2 = 1$, namely, under the QWS condition, one could observe a quarter-cycle oscillation inside cladding layers $a$ and $b$, as exemplified in the Bragg fiber [25]. The odd multiple of $\pi$ in the lateral propagation constant times the cladding layer thickness also means that cladding fields show the maximum or minimum value in each interface, being consistent with the condition of outgoing flux minimization [2].

A generalized Bragg fiber consisting of three kinds of cladding materials has been presented [26]. It was also confirmed for the generalized Bragg fiber that the in-phase condition at the core–cladding boundary yields the antiresonant reflection and the generalized QWS conditions in a way similar to the case of two-layer types (such as a Bragg fiber).

### 3. EIGENVALUE EQUATIONS AND CLADDING THICKNESSES UNDER THE GENERALIZED QWS CONDITION IN BRAGG FIBER

It was confirmed in Section 2 that the in-phase condition at the core–cladding boundary is equivalent to the generalized QWS condition (and, accordingly, the antiresonance condition) in the Bragg fiber. We derive eigenvalue equations under the generalized QWS condition next.

Eigenvalue equations of a general case have been derived for the TE, TM, and hybrid modes in Eqs. (21) and (26) of [5] in the framework of asymptotic expansion approximation, where $N = \infty$ in Fig. 1. They are reconsidered here under the generalized QWS condition [Eq. (4)]. Main parameters can be represented by

$$X_S = \frac{1}{2} \left( \tilde{\zeta}_b \kappa_b + \frac{\zeta_a \kappa_a}{\zeta_b \kappa_b} \right) (-1)^{q_2}, \quad (S = \text{TE or TM}), \quad (7a)$$

$$Y_S = \frac{1}{2} \left( \tilde{\zeta}_b \kappa_b + \frac{\zeta_a \kappa_a}{\zeta_b \kappa_b} \right) (-1)^{q_2}, \quad (7b)$$

$$\exp(-iK_j^2 \lambda) = \text{Re}(X_S) \pm \sqrt{\left| \text{Re}(X_S) \right|^2 - 1}^{1/2}$$

$$= \frac{(-1)^{q_2}}{2} \left( \frac{\zeta_a \kappa_a}{\zeta_b \kappa_b} \right)^{1/2} \left( \frac{\tilde{\zeta}_b \kappa_b - \zeta_a \kappa_a}{\tilde{\zeta}_b \kappa_b} \right) \left( j = 1, 2 \right). \quad (7c)$$

$$\zeta_i = \left\{ \begin{array}{ll} 1/n^2_i & \text{for } S = \text{TE} \\ 1/n^2_i & \text{for } S = \text{TM}. \end{array} \right. \quad (7d)$$

Here, $K_j^2$ denotes the Bloch wavenumber, and $S$ in the subcript and superscript applies to TE or TM, respectively. The upper and lower signs in the double sign notation correspond to $j = 1$ and $j = 2$, respectively, in Eq. (7c). Bloch wavenumbers, $K_1^2$ and $K_2^2$, are valid only for odd and even values of $q_2$, respectively.

Eigenvalue equations of the TE, TM, and hybrid modes under the generalized QWS condition can be summarized as

$$\kappa_a x_c = 2 \pi r_0 \left[ \frac{\beta}{\kappa_b} \right]^{1/2} = U_{\text{QWS}} \quad (8a)$$

with

$$U_{\text{QWS}} = \begin{cases} J_{j, \mu} = j_{j, \mu + 1} & : \text{TE}_{\mu} \text{ mode} \\ J_{0, \mu} = 0 & : \text{TM}_{\mu} \text{ mode} \\ J_{j, \mu} (\nu \geq 1) = \text{HE}_{\mu} \text{ mode} \\ J_{j, \mu} (\nu \geq 1) = \text{EH}_{\mu} \text{ mode} \end{cases} \quad (8b)$$

in a manner similar to that under the QWS condition [5]. Here, $J_{j, \mu}$ and $j_{j, \mu}$ indicate the $\mu$th zeros of Bessel function $J_\nu$ of order
\( \nu \) and its derivative \( J'_\nu \) with respect to the argument, respectively. \( \nu \) and \( \mu \) stand for the azimuthal and radial mode numbers, respectively. The above result is valid for \( n_a > n_b > n_c \), and \( n_a \geq \sqrt{2} n_c \) for the TE and TM modes, respectively. Note that \( q_2 \) is included in Eqs. (7a)–(7c) but not in Eq. (8a).

Equation (8a) is in formal agreement with that under the QWS condition \([5]\) whether \( q_2 \) is odd or even. Equation (8a) implies that the eigenvalue equation can be obtained from the core parameters alone. If \( U_{\text{QWS}} \) in Eq. (8a) is replaced by \( U_{\text{QWS}} = \lambda_{a,b} \) \([12]\), which is introduced in the limit of an infinite \( V \) parameter in the ARROW model, then we can obtain an expression identical to that in Eq. (8a). This agreement in the eigenvalue equation is because both cases are valid for an infinitesimal value of wavelength \( \lambda_0 \).

Under the generalized QWS condition, waves reflected from each cladding interface contribute in phase to that at the core–cladding boundary, as stated above. Consequently, waves within the core behave as if the cladding is not structured, and a standing wave arises. Then, the core–cladding boundary acts as the node or antinode, which is why \( U_{\text{QWS}} \) takes on \( \lambda_{a,b} \) or \( \lambda \).

By omitting \( \beta \) from Eq. (1) and using Eq. (8a), one obtains an expression of the lateral propagation constant as

\[
\kappa_i = \left[ \left( n_i^2 - n_0^2 \right) k_0^2 + \left( \frac{U_{\text{QWS}}}{r_e} \right)^2 \right]^{1/2} \quad (i = a, b). \tag{9}
\]

Combination of Eq. (4) with Eq. (9) leads to the cladding layer thicknesses \( a \) and \( b \) under the generalized QWS condition as

\[
a = \frac{q_1 - 1/2}{2} \left[ n_a - n_b^2 + \left( \frac{U_{\text{QWS}}}{2 \pi r_e} \right)^2 \right]^{-1/2}, \tag{10a}
\]

\[
b = \frac{q_2 - q_1 + 1/2}{2} \left[ n_b - n_c^2 + \left( \frac{U_{\text{QWS}}}{2 \pi r_e} \right)^2 \right]^{-1/2}. \tag{10b}
\]

If we set \( q_1 = q_2 = 1 \) in Eqs. (10a) and (10b), then these expressions reduce to prior results under the QWS condition \([5]\). It is found from Eq. (10a) that the cladding layer thickness \( a \) can be increased by increasing the value of \( q_1 \), and similarly for the cladding layer thickness \( b \) with respect to both \( q_1 \) and \( q_2 \) with the remaining parameters unchanged.

Omitting \( (U_{\text{QWS}}/2 \pi r_e)^2 - n_e^2 \), core information, from Eqs. (10a) and (10b) we have an expression of the wavenumber as

\[
k_{\text{QWS}} = \pi \left\{ \frac{1}{n_a^2 - n_b^2} \left( \frac{q_1 - 1/2}{a} \right)^2 - \left( \frac{q_2 - q_1 + 1/2}{b} \right)^2 \right\}^{1/2}. \tag{11}
\]

The \( k_{\text{QWS}} \) is in an exact accordance with \( k_c \), representing the central gap point \( P_c = (k_c, n_c) \) \([24]\), if we set \( q_1 = m_1 \) and \( q_2 - q_1 = m_2 \). In addition, the substitution of Eq. (1) into Eq. (4), omission of the wavelength from them, and introduction of the effective modal index \( \tilde{n} = \beta / k_0 \) produces

\[
\tilde{n}_{\text{QWS}} = \left( \frac{n_a^2 - n_b^2 n_c^2}{1 - n_e^2} \right)^{1/2} \tag{12}
\]

with \( n_e \equiv [(q_1 - 1/2)b]/[(q_2 - q_1 + 1/2)a] \). The \( \tilde{n}_{\text{QWS}} \) exactly agrees with \( n_e \) of the \( P_c \). The \( P_c \), which is derived from the antiresonance condition, provides a simple means of determining the corresponding \( (k, \tilde{n}) \) in the SPARROW model \([24]\).

Equations (11) and (12) show an analytical equivalence between the present generalized QWS condition and the central gap point \( P_c \) and consist of the cladding parameters alone. On the other hand, Eqs. (10a) and (10b) include both core and cladding parameters, and Eq. (8a) means that the effective modal index \( \tilde{n} \) can be calculated from the core and mode parameters.

### 4. TRANSFORMATION OF ANTiresONANT REFLECTION INTO IN-PHASE CONDITIONS IN GENERAL MEDIA

#### A. Phase in Structure Combined by Two Adjoining Layers Satisfying the Antiresonant Reflection Condition

Let us consider a structure where layers \( a \) and \( b \) are combined. The structure can be classified into two cases depending on the refractive index distribution, as shown in Fig. 3. It is assumed that two adjoining layers satisfy the antiresonant reflection condition. The phase change for each round trip (within each layer) can thus be expressed as \([12,24]\)

\[
\Phi^a = 2\pi p_1 + \pi, \quad \Phi^b = 2\pi p_2 + \pi \quad (p_1, p_2: \text{integers}). \tag{13}
\]

We express the phase change due to the round trip in a combined structure as \( \Phi^a_{a+b} \). The value of \( \Phi^a_{a+b} \) depends on the refractive indices of layers that are next to layers \( a \) and \( b \). For a combination of refractive indices of both sides in Fig. 3(a) (see A), we can express the phase change as

\[
\Phi^a_{a+b} = 2\kappa_a a, \quad \Phi^b_{a+b} = 2\kappa_b b + \pi \tag{14}
\]

in terms of the lateral propagation constant and the thickness. Equating expressions in Eq. (13) to those in Eq. (14), we have expressions for \( \kappa_a a \) and \( \kappa_b b \). Then, the phase change in the combined structure is obtainable as

\[
\Phi^a_{a+b} = 2(\kappa_a a + \kappa_b b) = 2\pi(p_1 + p_2 + \pi). \tag{15}
\]

![Fig. 3. Color online Antiresonant reflection condition in a structure consisting of two layers with each satisfying the antiresonant reflection condition. (a) \( n_a > n_c \), (b) \( n_b > n_c \). Letters A and B (C and D) on the left side of layer A indicate a case where their refractive index (indicated by dotted lines) is higher (lower) than \( n_a \), while A and C (B and D) on the right side of layer B indicate a case where their refractive index is higher (lower) than \( n_b \). Letters A to D label combinations of refractive indices of layers situated outside of layers \( a \) and \( b \).](image)
being identical to the antiresonant reflection condition. Similarly, for the other combinations [B to D in Fig. 3(a)], we have the same expressions as the last expression in Eq. (15).

Equation (15) means that if two adjoining layers satisfy the antiresonant reflection condition, the combined structure also satisfies the antiresonant reflection condition. This is because a round trip wave traverses in opposite directions at the interface of layers \(a\) and \(b\), and the phase change due to the reflection is canceled out there. We can also obtain a result identical to Eq. (15) for the structure shown in Fig. 3(b).

In summary, if a structure consists of plural layers, each of which is in contact with one another and satisfies the antiresonant reflection condition individually, then the structure also satisfies the antiresonant reflection condition as one unit. The equivalence of the in-phase condition with the antiresonance condition implies that the results of previous works, such as [12, 24], are still wholly valid, but were just based on a slightly more simplistic physical approximation.

B. Transformation of Antiresonant Reflection into In-Phase Conditions

Assume that cladding layer \(a\) satisfies the antiresonant reflection condition and that its refractive index is always higher than that of the inner side layer, which is usually the core (see Fig. 4). This situation can apply to the Bragg and the ARROW-type fibers.

In a case where layer \(a\) satisfies the antiresonant reflection condition, the phase change at each round trip can be represented by

\[ \Phi_a^{rt} = 2\pi p + \pi \quad (p: \text{integer}). \]  (16)

For cases where the refractive index of a layer outside of layer \(a\) is higher than that of layer \(a\) (case A in Fig. 4), one obtains

\[ \Phi_a^{rt} = 2\kappa_a a + \pi. \]  (17)

The phase change caused by the reflected wave becomes

\[ \Phi_a = 2\kappa_a a = 2\pi p \]  (18)

at the core–cladding boundary. Similarly, we obtain \( \Phi_a = 2\pi p \) for case B in Fig. 4.

The \( \Phi_a \) in Eq. (18) is an integer multiple of \( 2\pi \), being similar to Eq. (3). Hence, we can say that if a layer satisfies the antiresonant reflection condition, then the wave reflected from the outer interface always satisfies the in-phase condition at the core–cladding boundary.

Cladding layer thickness will be evaluated for the antiresonant reflection condition in the following way. For the high-index region in Fig. 3(b) or Fig. 4, we have \( 2\kappa_a a = 2\pi p + \pi \) from Eqs. (16) and (17). Equating \( \kappa_a \) obtained from this equation to that from Eq. (9), we have an expression of cladding thickness \( a \) where we set \( q_1 = p_1 \) in Eq. (10a). For the low-index region in Fig. 3(b), one obtains \( 2\kappa_a b = 2\pi p_2 + \pi \). Similarly, we have an expression of cladding thickness \( b \) where we set \( q_2 - q_1 = p_2 \) in Eq. (10b). The expression concerning \( p_1(m_1) \) and \( p_2(m_0) \) has been presented for the ARROW [12] and SPARROW models [24].

It is concluded from the discussions of Sections 2 and 4 that the antiresonant reflection condition is equivalent to the in-phase condition at the core–cladding boundary in the 1D Bragg waveguide. The 1D result also holds for the cylindrically symmetric 2D structure, as stated in Section 2. The distinction between the Bragg fiber and the ARROW model is summarized as follows. As for the waveguide structures, the Bragg fiber requires the periodicity of cladding, while the ARROW model makes it possible to design each cladding layer individually. As for the guiding mechanisms, the Bragg fiber makes use of the Bragg diffraction in the cladding and is based on the in-phase condition, while the ARROW model utilizes the antiresonant reflection condition for each cladding layer. Accordingly, the ARROW model has the freedom of fiber design but is complicated in the theoretical analysis of propagation properties. On the other hand, the periodicity in the Bragg fiber has the merit of using the Bloch theorem, which enables us to explicitly derive eigenvalue equations.

5. USEFULNESS OF GENERALIZED QWS CONDITION

Usefulness of the generalized QWS condition is exemplified next by showing the CL of the hollow-core Bragg fiber. We prescribe \( n_a = 2.5, \ n_c = n_{ex} = 1.5, \ n_e = 1.0, \ r_e = 2.0 \mu m, \) and \( \lambda_0 = 1.0 \mu m \) in Fig. 1 for all combinations of \( q_1 \) and \( q_2 \) that follow. Cladding layer thickness \( b \) is first prescribed for all cases by setting \( p_2 = q_2 - q_1 = 0 \) in Eq. (10b). Next, layer thickness \( a \) is prescribed to satisfy Eq. (10a) for \( p_1 = q_1 = 5 \). Then, we have \( a = 0.973 \mu m, \ b = 0.216 \mu m, \) and \( \Lambda = 1.189 \mu m \) for the TE\(_{01}\) mode. For number of cladding pairs \( N = 5 \), we have \( \Lambda a = 5.946 \mu m, \) the thickness of periodic cladding, on which the CL strongly depends. Hence, for comparison with various combinations of \( p_1 = q_1, N \) must be chosen such that \( \Lambda a \) agrees with each other as much as possible. For example, for \( p_1 = q_1 = 1 \) of the TE\(_{01}\) mode, one obtains \( a = 0.108 \mu m, \ \Lambda = 0.324 \mu m, \) and \( N = 18, \) and \( \Lambda a = 5.830 \mu m, \) as shown in Table 1, where an integer value of \( N \) was determined by rounding. The CL is computed using the multilayer division method [3], which makes use of no approximation in its formalism and, hence, has a high accuracy comparable to that of the transfer matrix method [2]. The CL will be normalized by the \( \Lambda a \) for later use.

The normalized CL is shown in Fig. 5(a) as a function of various \( p_1 = q_1 \) with \( p_2 = 0 \) for five lower-order modes, whose fiber parameters are set in a way described earlier. We can find that the CL shows minima at integers of \( p_1 = q_1 \) for

![Fig. 4](color online) Case where layer \( a \) satisfies the antiresonant reflection condition, and its refractive index \( n_a \) is higher than the index \( n_c \) of its inner side layer. Letters A and B on the right side of layer \( a \) indicate a case where its refractive index (indicated by dotted lines) is higher and lower than \( n_a \), respectively.
lower-order modes. This implies that the CL takes on minima for the generalized QWS condition introduced in Section 2 and for the central gap point [24]. The minimum CL values show the lowest value for the TE\textsubscript{01} mode, as shown in [8,9], and they increase with increasing the integer \(p_1 = q_1\). There are no values for \(p_1 = q_1 < 0.5\) because the cladding layer thickness \(a\) becomes negative there. The tendency of CL on the \(p_1 = q_1\) similar to that above was also confirmed for different cladding index contrasts, namely, \(n_a = 3.5\).

Figure 5(b) illustrates the normalized CL as a function of \(q_2\) with \(q_1 = 1\) for the TE\textsubscript{01} mode. Cladding layer thicknesses \(a\) and \(b\) are prescribed to satisfy Eqs. (10a) and (10b). The number \(N\) of cladding pairs for various \(q_2\) is set such that the thickness of the periodic cladding \(a\) is nearly identical with that for \(q_2 = 5\) and \(N = 3\), in which \(a = 0.108 \mu m\), \(b = 1.942 \mu m\), \(\Lambda = 2.050 \mu m\), and \(N\Lambda = 6.149 \mu m\). The CL exhibits minima and maxima at nearly integers and near half-integers, respectively, of \(q_2\). The minimum CL values increase with the increase in integer \(q_2\). The lowest CL value is obtained at \(q_2 = 0.99\). This slight shift from the integer is due to the fact that \(N = 18\) is used here for \(q_1 = q_2 = 1\), while Eqs. (10a) and (10b) are derived for \(N = \infty\). It can be seen from Figs. 5(a) and 5(b) that the CL shows the lowest value at \(q_1 = 1\), \(q_2 \cong 1\), the QWS condition.

Although numerical methods, such as the transfer matrix, multilayer division, and finite-difference time domain methods, can evaluate the CL for general cases, they can not explicitly predict the operation condition required for the minimum CL before performing numerical computations. Contrary to the numerical methods, the present phase theory as well as the SPARROW model [24] makes it possible to explore significant discrete points, at which the CL shows its minima, in a simple calculation.

### 6. APPLICATION OF PHASE THEORY TO UNDERSTANDING PBG DEPENDENCE OF LOSSES IN PBFS

Let us consider a fiber that has a cladding region with a refractive index higher than the core’s, such as a Bragg or ARROW-type fiber. These fibers show unique properties of confinement and bend losses. The properties can be explained using discussions described earlier.

#### A. Considerations of Phase

Assume that this kind of fiber is designed to satisfy the generalized QWS condition, which is the antiresonant reflection condition, at \(\lambda = \lambda_{QWS} = \lambda_{anti} = 2\pi/k_{QWS}\), where wavenumber \(k_{QWS}\) is determined to satisfy the QWS condition. After all fiber parameters are kept fixed, let the fiber be operated at

\[
\begin{align*}
\lambda_t &= \lambda_{QWS}/t = \lambda_{anti}/t, \\
k_t &= \kappa_{QWS} (t: \text{positive integer}).
\end{align*}
\]

Substitution of Eq. (19) into Eq. (1) leads to an expression of the lateral propagation constant:

\[
\kappa_{it} = k_t[\nu_i^2 - (\beta/k_t)^2]^{1/2} \quad (i = a, b).
\]

It was confirmed by numerical calculation that values of \((\beta/k_t)\) are nearly unchanged despite \(t\) (corresponding to wavelength) for a sufficiently confined case. The \(\kappa_{it}\) can thus be approximated by

\[
\kappa_{it} \cong \nu_i k_t. \tag{21}
\]

Let a wave of wavelength \(\lambda_t\) be reflected by the outer interface of the \(m\)th cladding layers \(a\) and \(b\). Then, phase changes at the core–cladding boundary can be represented by

\[
\Phi_{a,n} = 2[\kappa_{at}a + (m-1)(\kappa_{at}a + \kappa_{bt}b)] + \pi \\
\cong 2\pi (q_1 + (m-1)q_2) - \pi t + \pi. \tag{22a}
\]

![Image](image_url)
The value of $\Phi_{b,m}^l$ is approximately an integer multiple of $2\pi$ for an odd $l$ and is an odd multiple of $\pi$ for an even $l$. The value of $\Phi_{a,m}^l$ is approximately an integer multiple of $2\pi$ regardless of $l$. For $l = 1$, the $\Phi_{a,m}^l$ and $\Phi_{b,m}^l$ in Eqs. (22a) and (22b) agree with the $\Phi_{a,m}^l$ and $\Phi_{b,m}^l$, respectively, in Appendix A.

Therefore, waves reflected from each cladding interface become in phase just inside the core-cladding boundary for an odd $l$ and out of phase for an even $l$. It is expected that the optical confinement would be excellent for an odd $l$ and not so good for an even $l$ as a result of the destructive interference. This fact is reflected on the wavelength dependence of confinement and bend losses, as explained in Subsection 6.B.

When a wave of wavelength $\lambda_l$ travels back and forth inside cladding layer $a$ or $b$, their phase changes at the round trip become

$$
\Phi_{a}^l = 2\kappa_{d} a + 2\pi \cong 2\pi(lp_1 + 1) - \pi t,
\Phi_{b}^l = 2\kappa_{d} b \cong 2\pi tp_2 + \pi t
$$

(23)

with the aid of $2\kappa_d a + 2\pi = 2\pi p_1 + \pi$ and $2\kappa_d b = 2\pi p_2 + \pi$ for the ARROW condition.

For an odd $l$, both $\Phi_{a}^l$ and $\Phi_{b}^l$ are approximately odd multiples of $\pi$ and satisfy the ARROW condition. Conversely, for an even $l$, both $\Phi_{a}^l$ and $\Phi_{b}^l$ are approximately even multiples of $\pi$ and satisfy the resonant condition. This result is equivalent to that obtained by the in-phase condition. It is also valid for a case where cladding layers $a$ and $b$ are replaced by high- and low-index regions.

Consider a microstructured optical fiber (MOF) that has high-index inclusions in the cladding [14]. Cylindrical inclusions with diameter $d$ and index $n_b$ are embedded in the background material with an index $n_1 (n_b > n_1)$. The MOF satisfies

$$
\lambda_{\text{ant}}^{\text{cyl}} = \frac{2d(n_b^2 - n_1^2)^{1/2}}{p} \quad (p = 1, 2, 3, \ldots)
$$

(24)

for the antiresonant condition. For a small index contrast, the $\lambda_{\text{ant}}^{\text{cyl}}$ in Eq. (24) is approximated by

$$
\lambda_{\text{ant}}^{\text{cyl}} \cong \frac{2n_b d \sqrt{2}\Delta}{p}
$$

(25)

with the relative index difference $\Delta$.

For a wave satisfying the antiresonant condition, we have a relation $\kappa_{\text{ant}} d / \pi = p$ from Eqs. (C1) and (24), where $\kappa_{\text{ant}}$ denotes the lateral propagation constant at $\lambda_{\text{ant}}^{\text{cyl}}$. The resonant condition is represented by (see Appendix C)

$$
\lambda_{\text{res}}^{\text{cyl}} = \frac{2d(n_b^2 - n_1^2)^{1/2}}{p + 1/2} \quad (p = 1, 2, 3, \ldots)
$$

(26)

with the help of

$$
p \equiv \nu + 2\mu - 1.
$$

(27)

The $\lambda_{\text{res}}^{\text{cyl}}$ agrees with $\lambda_{m}$ in Eq. (2) of [14], if $p$ in Eq. (27) corresponds to $m$. In the case of a fiber consisting of a layer of thickness $d$, it has numerically been confirmed [14] that the antiresonant $\lambda_{\text{ant}}^{\text{cyl}}$ and resonant wavelengths $\lambda_{\text{res}}^{\text{cyl}}$ are obtained by interchanging between Eqs. (24) and (26), where $p = 0, 1, 2, \ldots$ for $\lambda_{\text{ant}}^{\text{cyl}}$.

The phase defined in Eq. (C2) becomes

$$
\phi_{\text{ant}} = \mu \pi - 3\pi/4 \quad \text{for the antiresonant condition}
$$

(28a)

and

$$
\phi_{\text{res}} = \mu \pi - \pi/2 \quad \text{for the resonant condition} \quad (\mu: \text{integer}).
$$

(28b)

Values of $\phi_{\text{ant}}$ are substantially $\pi/4$ or $5\pi/4$ for the antiresonant condition, whereas values of $\phi_{\text{res}}$ are $\pi/2$ or $3\pi/2$ for the resonant condition.

It is assumed that the MOF is designed at a wavelength of $\lambda_{\text{ant}}^{\text{cyl}}$ under fixed values of fiber parameter. Let the fiber be operated at $\lambda_l$ as shown in Eq. (19). Then, the lateral propagation constant $\kappa_{\text{eff}}$ at $\lambda_l$ is written as $\kappa_{\text{eff}} \equiv \ell \kappa_{\text{ant}}$, being similar to that in Eq. (21). The phase for $l$ can be represented by

$$
\phi_l = \pi tp / 2 - \pi \nu / 2 - \pi / 4
$$

(29)

using the phase defined in Eq. (C2).

Behavior for $l$ can be judged from consideration on $\phi_l$. In the case of $p = 1 (\nu = 0, \mu = 1)$ and $p = 3 (\nu = 0, \mu = 2$ or $\nu = 2, \mu = 1)$, we have the antiresonant behavior only for an odd $l$. For $p = 2 (\nu = 1, \mu = 1)$ and $p = 4 (\nu = 1, \mu = 2$ or $\nu = 3, \mu = 1)$, we always obtain the antiresonant behavior despite $l$. The antiresonant behavior indicates the maintenance of high optical confinement to the core. In contrast, for an even $l$ of $p = 1$ and $p = 3$, the resonant condition holds, and the optical confinement is extremely reduced.

This subsection presented three ways for understanding the dependence of optical confinement on $l$ by taking the phase into consideration.

B. Discussions on Confinement and Bend Losses

For large $V$ parameter of conventional optical fibers, the optical confinement is improved and the bend loss becomes small [27]. This means semiquantitatively that the optical confinement closely relates to the bend loss. The Bragg diffraction, relating to the interference, contributes to the formation of the photonic band gap [28]. Hence, the phase theory described earlier can be applied to understanding the properties of confinement and bend losses, wavelength dependence of which is studied next.

Bend loss of the hollow-core Bragg fiber has been measured as a function of wavelength [10]. Its cladding consists of an As$_2$S$_3$cyl layer ($n_b = 2.8$) and a polyether sulphone film ($n_1 = 1.55$). Much power was transmitted for the fundamental PBG at 3.55 $\mu$m, whereas the fiber suffered from much bend loss for the second PBG at 1.7 $\mu$m. The core radius can be estimated to be 308 $\mu$m using data drawn from the same preform although it is not shown in the text. Although the thickness of the first and last layers $a$ is 135 nm, we make use of the dominant thickness $a = 270$ nm for the estimation below. For $a = 270$ nm, $b = 900$ nm, and $n_c = 1.0$, we have $p_1 = q_1 \equiv 0.898$ and $p_2 = q_2 - q_1 \equiv 0.100$, and, consequently, $q_2 \equiv 0.998$ with the help of Eqs. (10a) and (10b). This means that the QWS condition is satisfied for cladding layer $b$, while it is roughly satisfied for cladding layer $a$. Accordingly, bend loss is small for the fundamental PBG but large for the second PBG.
Although calculation was made for the TE$_{01}$ mode, the result is kept unchanged as far as a relatively lower-order mode is prescribed.

The CL has been evaluated by applying the antiresonant reflection condition to the Bragg fiber filled by fluid in the core [24]. Cladding materials are the same as those described just above and $N = 4$. The value of $\lambda_{\text{QWS}}$ was estimated from $1.259/(p + 1/2)$ using Eq. (26) with $n_b = n_a = 2.8$, $n_l = n_b = 1.55$, and $d = 270$ nm. For $p = 0$, we have $\lambda_{\text{QWS}}^{(0)} \approx 5.25$ $\mu$m. The core radius is $10$ $\mu$m, its index is $n_c = 1.45$, and $n_c/n_b = 0.995$. Substituting these parameters into Eqs. (10a) and (10b), we have $p_1 = q_1 \approx 1.014$ and $p_2 = q_2 - q_1 \approx -0.094$, and, consequently, $d = 0.920$. Since fiber parameters are set to shift from the QWS condition, the CL exhibits large loss for the second and fifth PBGs but not for the fourth PBG. The evaluated result is in good agreement with that by the SPARROW model [24]. The discrepancy between data and consideration in Subsection 6A is caused by a small number of cladding pairs, $N = 4$.

Wavelength dependence of the CL is shown in Fig. 6 for the TE$_{01}$ mode of Bragg fiber with $n_c = 1.0, 1.4$, in which cladding layer thicknesses $a$ and $b$ are prescribed to exactly satisfy the QWS condition, namely, $q_1 = q_2 = 1$ or $p_1 = 1$ and $p_2 = 0$ at $\lambda_{\text{QWS}} = 1.0$ $\mu$m in Eqs. (10a) and (10b). The number of cladding pairs is $N = 20$. Refractive indices and core radius are the same as those in Fig. 5 except for $n_c$. Material losses of all media are not considered. The $\ell = 1$ corresponds to the fundamental PBG. CL values of odd $\ell$ are extremely lower than those of even $\ell$ despite $n_c/n_b$. In the vicinity of even $\ell$, CL values at $n_c = 1.4$ are lower than those at $n_c = 1.0$, but still are markedly higher than those of odd $\ell$. For $n_c = 1.0$ and 1.4, the CL near $\ell = 1$ is lower by about 11 and 17 orders, respectively, than that near $\ell = 2$. Although the CL near $\ell = 1$ is slightly higher than those near other odd $\ell$, the width of the low loss region becomes narrow with increasing $\ell$. The CL shows the minimum value at wavelength $\lambda_{\text{min}}$, which slightly shifts from the $\lambda_{\text{QWS}}/\ell$ with odd $\ell$. The shifts for $n_c = 1.4$ are larger than those for $n_c = 1.0$. The shifts are because only the phase is taken into consideration in the present theory, and a change in the amplitude must be considered for improved accuracy.

A comparison of results of Fig. 6 with that of the above $N = 4$ supports the following assertion: if fiber parameters are prescribed to exactly satisfy the QWS condition at a certain wavelength, then an extremely low CL can be obtained for odd $\ell$. This is consistent with the performance based on the phase consideration (Subsection 6A). The shift of $n_c$ from the unity gives rise to a reduction of CL for even $\ell$.

Significant bend loss difference between odd- and even-numbered PBGs similar to that in Fig. 6 has been observed in an all-solid silica PBF [18]. The core and background consist of an undoped pure silica, and the high-index region is formed by germanium-doped silica rods. One obtains $d = 6.69$ $\mu$m from $d/\Lambda = 0.44$ and $\Lambda = 15.2$ $\mu$m, where $\Lambda$ denotes the lattice constant and $d$ denotes the diameter of the cylindrical rod. Substituting $d$, $n_l = 1.458$, and $\Lambda = 2.03 \times 10^{-2}$ into Eq. (25), we have a relation, $\lambda_{\text{QWS}}^{(l)} = 3.83/p$ $\mu$m. The central wavelength of fundamental PBG is estimated to be 3.75 $\mu$m because the wavelength of the third PBG is 1250 nm. It is estimated that this fiber is operated near $p = 1$, then relative error is about 5% (3.75/3.93 = 0.95). Thus, discussions of $\ell$ described just after Eq. (29) are consistent with the dependence of bend loss on the order of PBG. The PBG order dependence of bend loss has semiquantitatively been explained by a picture based on the strength of overlap between the modes of neighboring rods [18]. The same result as above has also been obtained by evaluating the critical bend radius [18].

The present phase theory as well as the SPARROW model [24] can selectively evaluate lower loss values of confinement and bend losses unlike [18]. The phase theory allows us to
understand the characteristics in the form of the interference, the fundamental physical phenomenon.

7. SUMMARY
Guiding conditions were studied for the Bragg and the ARROW-type fibers. This was done by calculating the phase change of waves reflected from the cladding interfaces. We showed equivalence in guiding principle between the in-phase condition at the core–cladding boundary and the antiresonant reflection condition at each round trip in the 1D and cylindrically symmetric 2D structures with periodic cladding. The in-phase condition is equivalent to a generalized QWS condition in these structures. The eigenvalue equations of the Bragg fiber under the generalized QWS condition are identical to those in the limit of an infinite V parameter in the ARROW model. The generalized QWS condition is formally equivalent to the central gap point in the SPARROW model.

It was confirmed that the CL shows its minima for the generalized QWS condition of the Bragg fiber. It is important that parameters of the Bragg fiber are designed such that they satisfy the QWS condition, under which the CL exhibits the minimum value. The phase theory was applied to semiquantitatively understand the dependence of confinement and bend losses on the order of PBG in Bragg and ARROW-type fibers.

APPENDIX A: IN-PHASE CONDITION OF WAVES REFLECTED IN THE PERIODIC STRUCTURE
We consider the phase change \( \Phi_{i,m} \) \((i = a, b; m = 1, 2, 3, \ldots)\) of waves reflected from the outer interfaces of the \( m \)th cladding layers \( a \) and \( b \) (see Fig. 1). The in-phase condition for these waves is represented by

\[
\Phi_{a,m} = 2\kappa_a a + \pi + 2(m - 1)(\kappa_a a + \kappa_b b) = 2\pi q_{a,m}, \tag{A1}
\]

\[
\Phi_{b,m} = 2m(\kappa_a a + \kappa_b b) = 2\pi q_{b,m}, \tag{A2}
\]

where \( q_{a,m} \) and \( q_{b,m} \) are integers, and \( q_{b,m} \geq q_{a,m} \). Consideration on \( \Phi_{b,m+1} - \Phi_{b,m} \) produces

\[
\kappa_a a + \kappa_b b = \pi(q_{b,m+1} - q_{b,m}), \quad q_{b,m+1} - q_{b,m} = q_2 \tag{A3}
\]

with the aid of \( \Phi_b \) in Eq. (2). From these equations one obtains

\[
\kappa_a a = \pi( q_2 - q_{b,m} + q_{a,m} ) - \pi/2,
\]

\[
\kappa_b b = \pi(q_{b,m} - q_{a,m}) + \pi/2. \tag{A4}
\]

Above \( \kappa_a a \) and \( \kappa_b b \) are half-integer multiples of \( \pi \) regardless of \( m \).

From the requirement that \( \kappa_a a \) and \( \kappa_b b \) appearing in Eq. (A1) must agree with those in Eq. (4) in the periodic structure, we have

\[
q_{b,m} - q_{a,m} = q_2 - q_1. \tag{A5}
\]

Then, we can express \( \Phi_{a,m} \) and \( \Phi_{b,m} \) as

\[
\Phi_{a,m} = 2\pi(q_1 + (m - 1)q_2), \quad \Phi_{b,m} = 2\pi q_2. \tag{A6}
\]

Comparison of Eq. (A6) with Eqs. (A1) and (A2) gives

\[
q_{a,m} = q_1 + (m - 1)q_2, \quad q_{b,m} = mq_2. \tag{A7}
\]

Equations (A5) and (A7) mean that the accumulated phase changes are represented in terms of \( q_1 \) and \( q_2 \), which are integers defined for the first period.

APPENDIX B: CYCLES OF STANDING WAVE PATTERN
Under an asymptotic expansion approximation, cladding fields of the Bragg fiber can be represented by \( \exp(\pm i\kappa d/r) \) with the subscript \( i = a, b \) and the radial coordinate \( r \). For cladding layer thicknesses \( a \) and \( b \), one can observe a standing wave pattern of \( \kappa_a d/2\pi \) and \( \kappa_b d/2\pi \) cycles, respectively. Substitution of Eq. (4) into these expressions yields an expression of cycles described in the text.

APPENDIX C: DERIVATION OF RESONANT CONDITION IN EQ. (26)
The resonant condition of the MOF is derived from \( J_\nu(\kappa d/2) = 0 \) \([14]\) with an integer \( \nu \) and the lateral propagation constant defined by

\[
\kappa_d \equiv (2\pi/\lambda_b)(n_b^2 - n_f^2)^{1/2} \tag{C1}
\]

glancing angles of incidence. A cosine approximation of the Bessel function \( J_\nu(z) \) is given by \( (2/\pi z)^n \cos[z - (2\nu + 1)\pi/4] \) for \( z \gg \nu \). Then, the phase is defined by

\[
\phi \equiv \kappa_d d/2 - \nu \pi/2 - \pi/4. \tag{C2}
\]

A phase of the resonant condition can be written as \( \phi_{\text{res}} = \mu \pi - \pi/2 \) with an integer \( \mu \). Equating the \( \phi_{\text{res}} \) to Eq. (C2) and utilizing \( \kappa_d \), we obtain the expression in Eq. (26). Here, a new parameter \( p \) was introduced such that Eq. (26) is in accordance with the expression in [14].

REFERENCES