Confinement loss, including cladding material loss effects, in Bragg fibers

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Confinement loss (CL), including cladding material losses, is comprehensively evaluated for TE, TM, and hybrid modes of hollow-core Bragg fibers. We show that the combined fiber loss tends to approach a loss depending only on the CL, which is caused by the finite thickness of the periodic cladding region [1], yielding an insufficient optical confinement due to the Bragg diffraction originating from the periodicity in the cladding. The CL is a key factor for the appropriate design of PBGFs because periodic cladding must be as thin as possible to reduce the fabrication process. Confinement and cladding material losses are closely related.

PBGFs are primarily classified into two categories: one is a hollow-core PBF with air holes in the cladding and the other is a Bragg fiber that consists of an air core surrounded by periodic cladding with alternating high and low refractive indices. A low-loss PBGF has been fabricated using a seven-cell core [2]. The CL of hollow-core PBGFs has been investigated using a full-vector finite-element method (FEM) [3]. The antiresonant feature of an antiresonant reflecting optical waveguide has been used as a core surround in hollow-core PBGFs to reduce the CL [4].

The Bragg fiber has been fabricated using a chalcogenide glass polymer [5], an air-silica ring [6], and an air-polymer ring structure [7]. The CL in the Bragg fiber has been evaluated by the transfer matrix method [8], Chew’s method [9,10], a mixed method of exact and asymptotic analyses [11], and a multilayer division method [12] that was developed to apply a low-index contrast [13,14] to high-index-contrast fibers. The influence of surface mode on the CL spectrum was presented in the Bragg fiber with a bridge structure using FEM [15]. Previous papers barely considered cladding material losses except for fibers with a Si/Si3N4 structure [11]. On the other hand, the material loss dependence of fiber loss has theoretically been estimated [16] for the TE01 mode, which has the lowest loss [8,11,12] among all modes, by neglecting the field decay due to material loss.

The purpose of the present paper is to offer a comprehensive study on the CL of various modes of hollow-core Bragg fibers with cladding material losses by taking field decay into consideration. The CL, which can be calculated from the imaginary part of the propagation constant, is numerically evaluated here using the multilayer division method [12], where material losses are treated in terms of the complex refractive index.

This paper proceeds as follows. Section 2 shows the preparation of CL evaluation, including a brief description of the Bragg fiber. Section 3 verifies the results obtained by the present method and investigates the loss coefficient dependence of fiber loss. Section 4 presents numerical results on the CL of various modes to elucidate its properties. In Section 5, we present the numerical results of the TE01 mode in detail. Finally, Section 6 studies the PBG dependence of CL that exhibits a unique property.

1. INTRODUCTION

Hollow-core photonic bandgap (PBG) fibers (PBGFs) have the potential to achieve a loss lower than conventional silica fibers. Intrinsic optical loss factors include absorption, confinement, scattering, and structural imperfection losses, and extrinsic factors include bend and splicing losses. The confinement loss (CL) is caused by the finite thickness of the periodic cladding region [1], yielding an insufficient optical confinement due to the Bragg diffraction originating from the periodicity in the cladding. The CL is a key factor for the appropriate design of PBGFs because periodic cladding must be as thin as possible to reduce the fabrication process. Confinement and cladding material losses are closely related.

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2. PREPARATION OF CL EVALUATION

A. Brief Description of a Bragg Fiber

The Bragg fiber has a cylindrically symmetric microstructure with a hollow core surrounded by periodic cladding (Fig. 1). The core index is na and its radius is rc. The cladding has a finite number N of layer pairs that consist of high nha and low indices nhb(na > nb > nc). Their corresponding layer thicknesses are a and b, and the cladding period is Λ = a + b. The index of the cladding external layer is set to nce. The intensity absorption coefficients are designated by αa and αb for cladding layers a and b. The loss coefficient of the external layer is not considered to concentrate on the effect of cladding loss.
B. Calculation Method of CL with Cladding

Material Losses

Assume that the spatiotemporal dependence of the electromagnetic fields is represented by $U_{12} = \exp[i(\omega t - \beta z)]$, where $\omega$ and $\beta$ are the angular frequency and propagation constant, respectively. The propagation constant may be complex: $\beta = \beta_R + i\beta_I$. The CL, which arises from the finite thickness of the periodic cladding, can be evaluated from the imaginary part $\beta_I$ of the propagation constant $\beta$ as

$$\text{CL}[\text{dB/km}] = -\frac{20}{\ln 10} \times 10^{\beta_I [\mu\text{m}^{-1}]}.$$  

(1)

Here, we require $\beta_I < 0$ in the present framework.

Losses of cladding material are represented by complex refractive index $n_i$ as

$$n_i = n_i - i\kappa_i, \quad \kappa_i = \frac{\alpha_i c}{2\omega} (i = a, b),$$  

(2)

where $n_i$ denotes the normal index, $\kappa_i$ is the extinction coefficient of the cladding material, $\alpha_i$ is the intensity absorption coefficient of the cladding material, and $c$ is the light velocity in the vacuum. Subscripts $a$ and $b$ indicate cladding layers $a$ and $b$. Loss coefficient $\alpha_i$ is $\alpha_i [\text{dB/km}] = 10^\log(\epsilon) \times \alpha_i [\text{m}^{-1}]$. Equation (2) is available for $\alpha_i \ll 10^{10}[\text{dB/km}]$ at wavelength $\lambda_0 = 1.0 \mu\text{m}$ and for $\alpha_i \ll 10^{7}[\text{dB/km}]$ at $\lambda_0 = 10.0 \mu\text{m}$.

If the periodic cladding of the Bragg fiber extends to infinity, its eigenvalue equation can be simplified using an asymptotic expansion approximation [17]. However, if the periodic cladding is finite, we must resort to troublesome numerical means to investigate its electromagnetic properties. For a finite number $N$ of cladding pairs, we utilize the multilayer division method [12] to evaluate $\beta_I$ as well as $\beta_R$.

The multilayer division method, which is applied only to a cylindrically symmetric refractive index, is also available for the present case, if the normal index $n_i$ is formally replaced by the complex refractive index $\tilde{n}_i$. The restriction of application range yields a benefit that the number of parameters accompanied by the boundary condition is greatly reduced in comparison with versatile numerical methods, such as the FEM and the finite-difference time-domain method. The benefit is that the root finding is numerically stable even for the complex refractive index and higher-order modes. The mode of Bragg fibers with cladding material losses is easily designated (see Appendix A) near the quarter-wave stack (QWS) condition, which is important for practical use due to excellent optical confinement to the core.

C. Setting of Parameters

First, wavelength $\lambda_0$, refractive indices, and core radius $r_c$ are prescribed. Cladding thicknesses $a$ and $b$ are set to satisfy the QWS condition for the prescribed parameters as

$$\frac{a}{\lambda_0} = \frac{1}{2} \left[ 4 \left( n_a^2 - n_b^2 \right) + \left( \frac{U_{\text{QWS}}}{\pi} \right)^2 \left( \frac{\lambda_0}{r_c} \right)^2 \right]^{-1/2},$$  

(3)

which holds for a Bragg fiber with infinite periodic cladding [17]. $U_{\text{QWS}}$ is a constant peculiar to the mode and relates to the zeroes of the Bessel function. In addition, $b$ is obtained by an expression where $a$ and $n_i$ are replaced by $b$ and $n_i$ in Eq. (3).

Core index $n_c = 1.0$ and external layer index $n_{ex} = 1.5$ are used (except in Subsection 3.A) throughout this paper. The wavelength is set to $\lambda_{\text{QWS}} = 1.0 \mu\text{m}$ except for the data of Subsection 3.A and Section 6. We assume here that material loss coefficients have no wavelength dependence.

3. CONFIRMATION OF EVALUATED FIBER LOSS

A. Verification of Evaluated Fiber Loss Characteristics

CL, including cladding material losses, is first compared with those derived previously. Fiber parameters are prescribed to be the same as those in a theoretical estimation [16] for comparison: $\lambda_0 = 1.0 \mu\text{m}$, $n_a = 2.5$, $n_b = n_{ex} = 1.521$, $r_c = 1.1166 \mu\text{m}$, and $\alpha_a = \alpha_b = 10 \text{ dB/km}$.

The combined fiber loss is semilogarithmically plotted in Fig. 2 for the TE$_{01}$ mode of a hollow-core Bragg fiber as a function of $N$, the number of cladding pairs. The loss is reduced with increasing $N$ and converges to a constant value, $L_{\text{mat}} = 0.45 \text{ dB/km}$. This convergence results from the fact that the fields are almost confined within the core and the periodic cladding, and the loss depends on only the material loss limit value $L_{\text{mat}}$. The converged value is slightly higher than a previously estimated loss, 0.20 dB/km [16], because the present calculation considers the field decay due to cladding material losses, but [16] neglects it. For small $N$, the combined fiber loss agrees with $L_{\text{conf}}$, where only the CL is considered. For later use, a cross-point of the asymptotes of $L_{\text{mat}}$ and $L_{\text{conf}}$ is designated by $N_{\text{tr}}$, the transition point. $L_{\text{conf}}$ is dominant for

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Fig. 1. Schematics of Bragg fiber: $r_c$, core radius; $n_i$, refractive index of the core; $n_a$ and $n_b$, indices of layers with thicknesses $a$ and $b$; $\Lambda = a + b$, period in cladding; $n_{ex}$, external layer index; $N$, number of cladding pairs; $\alpha_a$ and $\alpha_b$, loss coefficients of cladding layers $a$ and $b$; $n_a > n_b > n_e$.

Fig. 2. Comparison of the combined fiber loss of the TE$_{01}$ mode with fiber loss derived without field decay. Solid curve, combined fiber loss; dashed line, fiber loss derived without field decay [16]; dotted-dashed line, CL alone [12]. $\lambda_0 = 1.0 \mu\text{m}$, $n_a = 2.5$, $n_b = n_{ex} = 1.521$, $r_c = 1.1166 \mu\text{m}$, and $\alpha_a = \alpha_b = 10 \text{ dB/km}$. 

$N < N_{tr}$ and $L_{mat}$ is dominant for $N > N_{tr}$. $L_{mat}$ is a measure of achievable minimum fiber loss, and transition point $N_{tr}$ is a measure of the $N$ needed to maintain the loss as low as possible.

For the TE$_{01}$ mode of a hollow-core Bragg fiber with four pairs of Si/Si$_3$N$_4$, the combined fiber loss was investigated by a mixed method of exact analysis and asymptotic expansion approximation [11]. Cladding parameters were $n_a = 3.5$ and $a = 0.11\mu m$ for Si and $n_b = 2.0$ and $b = 0.21\mu m$ for Si$_3$N$_4$. Other parameters were $\lambda_0 = 1.55\mu m$, $r_c = 7.5\mu m$, $n_c = n_{ex} = 1.0$, and $a_0 = 0 dB/km$. Only polysilicon loss $\alpha_b$ was considered in [11], which gave $1.2113 \times 10^5$ and $1.2123 \times 10^5$ dB/km for $\alpha_b = 10^5$ and $10^6$ dB/km, respectively. For corresponding $\alpha_b$, fiber losses of $1.1963 \times 10^5$ and $1.1973 \times 10^5$ dB/km were obtained by the present method that utilizes no approximation. The proportional coefficient of fiber loss was about $1.0656 \times 10^{-4}$ for the previous method, but it was $1.0668 \times 10^{-4}$ for the present method. Both methods provide reasonable agreement.

### B. Fiber Loss Dependence on Cladding Material Losses

Figure 3 illustrates the combined fiber loss of the TE$_{01}$ mode as a function of $N$ and various cladding material losses. Material losses were taken for 11 combinations of $\alpha_a$ and $\alpha_b$ up to $\alpha_a = \alpha_b = 10^5$ dB/km. Parameters were $\lambda_0 = 1.0$ $\mu$m, $n_a = 2.5$, $n_b = n_{ex} = 1.5$, and $r_c = 2.0$ $\mu$m. $L_{conf}$ is independent of the combinations of prescribed $\alpha_a$ and $\alpha_b$ values for small $N$. For a sufficiently small $N$, $L_{conf}$ obeys a formula:

$$L_{conf} \propto \left( \frac{a/b}{n_1^2 b/n_2^2 a} \right)^{2N} \quad \text{for TE mode}$$

$$L_{conf} \propto \left( \frac{a/b}{n_1^2 b/n_2^2 a} \right)^{2N} \quad \text{for TM and hybrid modes}$$

under the QWS condition [12]. As cladding material loss increases, $L_{mat}$ increases, but $N_{tr}$ decreases. If different values of $\alpha_a$ and $\alpha_b$ are interchanged, then the fiber loss with a small value of $\alpha_b$ is lower than that with a small $\alpha_a$. This is because cladding layer thickness $b$ is wider than layer thickness $a$ under the QWS condition. A condition $\alpha_a \geq \alpha_b$ is desirable for obtaining a low-loss fiber, if possible.

![Fig. 3. Cladding pair dependence of the combined fiber loss for the TE$_{01}$ mode as a function of the cladding material losses: $\lambda_0 = 1.0\mu m$, $n_a = 2.5, n_b = n_{ex} = 1.5$, and $r_c = 2.0\mu m$. $a = 0.1082\mu m$, $b = 0.2157\mu m$, and $\lambda = 0.3239\mu m$. Values shown in parentheses indicate cases in which $\alpha_a$ and $\alpha_b$ are different from each other.](image)

For example, one obtains a combined fiber loss of $7.2 \times 10^5$ dB/km for $\alpha_a = \alpha_b = 1.0 \times 10^6$ dB/km. This property is useful for obtaining a low-loss hollow-core Bragg fiber with such a large material loss as a polymer [7].

$N_{tr}$ decreases linearly with log $\alpha_a$ in Fig. 4. This relationship is derived as follows. Both $L_{mat}$ and $N_{tr}$ satisfy Eq. (4) at the transition point if the fiber parameters satisfy the QWS condition. From $N_{tr}$ for different $\alpha_a$ values, we obtain

$$\log[L_{mat}(\alpha_a)/L_{mat}(10)] = S [N_{tr}(\alpha_a) - N_{tr}(10)].$$

Here, $S$ indicates the slope in a semi-logarithmic plot and can be expressed as

$$S = \begin{cases} S_{TE} \equiv 2 \log(a/b), & \text{for TE mode} \\ S_{TM/TE} \equiv 2 \log (n_2^2 b/n_1^2 a), & \text{for TM and hybrid modes} \end{cases}$$

from Eq. (4). For example, $S = -5.99$ is obtained for the condition given in Fig. 3. The substitution of Eq. (5) into Eq. (6) leads to

$$N_{tr}(\alpha_a) = N_{tr}(10) + \frac{\log \alpha_a - 1}{S}.$$

![Fig. 4. Material loss limit value $L_{mat}$ and transition point $N_{tr}$ as a function of the cladding absorption coefficient. Fiber parameters are the same as those in Fig. 3: $\alpha_a = \alpha_b$.](image)
Equations (5) and (8) imply that the results for various combinations of $a_c$ and $a_b$ are predicted by a result for a particular combination of $a_c$ and $a_b$ for the same structure as long as the material losses are not so large. Note that these equations hold only for a fiber with identical fiber parameters. It is expected from these equations that we have $L_{mat} = 7.22 \times 10^4$ dB/km and $N_{\text{tr}} = 7.8$ for $a_c = a_b = 10^6$ dB/km under the fiber parameters shown in Fig. 3.

A linear relation between fiber loss and material loss holds even for $a_c = a_b$. We have $L_{mat} = 0.3689$ and $0.4256$ dB/km for two combinations of $a_c = 100$ dB/km and $a_b = 10$ dB/km and $a_c = 10$ dB/km and $a_b = 100$ dB/km, respectively. From these data, we can establish an empirical equation of material loss limit value

$$ L_{mat}(a_c, a_b) = C_a a_c + C_b a_b \quad (9) $$

with $C_a = 3.297 \times 10^{-3}$ and $C_b = 3.927 \times 10^{-3}$. Similarly, an expression of the transition point is given by

$$ N_{\text{tr}}(a_c, a_b) = N_{\text{tr}}(10, 10) + \frac{1}{S} \log \frac{L_{mat}(a_c, a_b)}{L_{mat}(10, 10)} \quad (10) $$

where $L_{mat}(a_c, a_b)$ in Eq. (9) is inserted into Eq. (10). Consequently, the combined fiber loss is presented for a certain combination of $a_c = a_b = 10$ dB/km, except as otherwise referred to, hereafter.

4. NUMERICAL RESULTS ON COMBINED FIBER LOSS OF VARIOUS MODES

A. Dependence on the Number of Cladding Pairs

Figure 5 semilogarithmically plots the dependence of the combined fiber loss on the number of cladding pairs for various modes. Fiber parameters are $\lambda_0 = 1.0 \mu m$, $n_a = 2.5$, $n_b = n_{ex} = 1.5$, $r_c = 2.0 \mu m$, and $a_c = a_b = 10$ dB/km. We plot all the $\text{TE}_{0\nu}$ and $\text{TM}_{b\mu}$ modes appearing at $\lambda_0 = 1.0 \mu m$, and $\text{HE}_{\nu\mu}$ and $\text{EH}_{\mu\nu}$ only with $\nu > \mu$ as (11), (12), and (21) to avoid complexity. As $N$ increases, the combined fiber loss of the individual modes decreases and converges to a constant value peculiar to the mode. The $\text{TE}_{01}$ mode in this case exhibits the lowest loss among all modes as well as for a case where only the CL is considered [8,12]; hence, both $L_{mat}$ and $N_{\text{tr}}$ are also the lowest in the $\text{TE}_{01}$ mode.

For a large value of $N$ in Fig. 5, the $L_{mat}$ of $\text{TM}_{b\mu}$ and $\text{EH}_{\nu\mu}$ modes hardly depends on the mode numbers, but that of $\text{TE}_{0\nu}$ and $\text{HE}_{\mu\nu}$ strongly depends on the mode numbers. This property can also be seen in the core radius dependence of the optical power confinement factor [18]. These properties well reflect the fact that the HE and EH modes respectively reduce to the TE and TM modes under the QWS condition [17]. The reduction of the HE and EH modes to the TE and TM modes could not be found when only the CL was considered [12]. For a small $N$, the combined fiber loss is nearly the same as the CL alone [12], whose slope can be represented by Eq. (7).

B. Characteristics of Material Loss Limit Value $L_{mat}$

The cladding index contrast dependence of material loss limit value $L_{mat}$ is shown in Fig. 6 for various modes. Fiber parameters are the same as those in Fig. 5 except for $n_a$. Thickness $a$ is determined to satisfy the QWS condition, Eq. (3), for each $n_a$. The number of cladding pairs is set to satisfy $N > N_{\text{tr}}$. $L_{mat}$ decreases smoothly with the increase in $n_a$ due to the improvement in optical power confinement.

Figure 7 is a logarithmic plot of material loss limit value $L_{mat}$ of various modes as a function of the core radius. Fiber parameters are the same as those in Fig. 5 except for $r_c$. Cladding thicknesses $a$ and $b$ are determined to satisfy the QWS condition for each $r_c$. An extremely large loss takes place at the lower limits of $r_c$, which correspond to the mode cutoff [17]. As $r_c$ increases, $L_{mat}$ markedly decreases for small $r_c$ and decreases at a constant rate for large $r_c$. The $L_{mat}$ of the TM and EH modes converges to an identical value despite mode numbers for large $N$. $L_{mat}$ decreases in inverse proportion to the third power of $r_c$ for the TE mode, whereas it decreases in a linear dependence on $r_c$ for the other three mode groups. A similar tendency has also been obtained in the CL [12] as well as in the optical power confinement factor [19,20] of the Bragg fiber. For $r_c \geq 0.8 \mu m$, the $\text{TE}_{01}$ mode shows the lowest loss.

A marked increase in $L_{mat}$ for small $r_c$ of Fig. 7 can be explained as follows. As $r_c$ decreases, each mode approaches the guiding limit of the mode, namely, the zero of real part $\beta_R$ of the propagation constant, and the optical power...
confinement is reduced, suffering readily from an influence of the cladding material losses.

C. Characteristics of Transition Point $N_{tr}$

The cladding index contrast dependence of transition point $N_{tr}$ is shown in Fig. 8 for various modes. The fiber parameters are the same as those in Fig. 5 except for $n_\alpha$. Thickness $a$ is determined to satisfy the QWS condition shown in Eq. (3) for each $n_\alpha$. $N_{tr}$ is reduced with increasing $n_\alpha$ because the optical power confinement is improved with increasing index contrast. Figure 8 shows that $N_{tr}$ for every mode approaches nearly close values with increasing cladding index contrast. $N_{tr}$ is less than about 34.8 for $n_\alpha \geq 2.5$ among the modes shown.

Figure 9 depicts the core radius dependence of transition point $N_{tr}$ for various modes. The fiber parameters are the same as those in Fig. 5 except for $r_c$. Cladding thicknesses $a$ and $b$ are prescribed based on the QWS condition for each $r_c$. As core radius $r_c$ increases, $N_{tr}$ decreases for the TE$_{0m}$ mode and increases for the TM$_{0m}$, HE$_{nm}$, and EH$_{nm}$ modes because fiber loss of the TE mode decreases faster than those of other modes for the increase in $N$ (Fig. 5). In addition, the $L_{mat}$ of the TE mode reduced faster than those of other modes for the increase in $r_c$ (Fig. 7). Hence, the $N_{tr}$ of only the TE mode decreases with the increase in $r_c$. This property is desirable for the single-mode transmission of the TE$_{0m}$ mode.

Figure 9 shows that the $N_{tr}$ value converges to a constant value depending on each mode regardless of the mode numbers with increasing $r_c$. The smallest converging $N_{tr}$ of 15.6 is obtained for the TE mode. The converging $N_{tr}$ values are 36.1, 34.8, and 34.8 for the TM, HE, and EH modes, respectively. For sufficiently large values of $r_c$, almost the whole power of each mode is confined to the center of the core. Then, the fields of each mode group are independent of mode numbers $\nu$ and $\mu$.

The lower limits of $r_c$ in $L_{mat}$ and $N_{tr}$ agree with each other for each mode (Figs. 7 and 9). As the core radius is reduced in Fig. 9, $N_{tr}$ converges to an identical value of about 21.8 except for the TM mode. This is because each mode approaches its guiding limit with decreasing $r_c$ and most fields extend to the cladding, resulting in no mode dependence of the fields. Therefore, there is no mode dependence for small $r_c$. The smallest $N_{tr}$ and $r_c$ of the TE$_{0m}$ mode are in accordance with those of the EH$_{10}$ mode with the same $\mu$. This reflects the fact that these modes degenerate under the QWS condition.

5. NUMERICAL RESULTS ON COMBINED FIBER LOSS OF THE TE$_{0m}$ MODE

We see from the results of Section 4 that the TE$_{0m}$ mode exhibits the lowest fiber loss even when cladding material losses are considered. Therefore, the characteristics of the TE$_{0m}$ mode will be scrutinized.

The dependence of the combined fiber loss of the TE$_{0m}$ mode is semilogarithmically plotted in Fig. 10 as a function of the number of cladding pairs, the cladding index contrast, and the core radius. The fiber parameters are $\lambda_0 = 1.0 \mu m$, $n_\alpha = n_{ex} = 1.5$, and $a_\alpha = a_0 = 10$ dB/km. Cladding layer thicknesses $a$ and $b$ are determined from Eq. (3) and a similar expression concerning $b$. As $N$ increases, the combined fiber loss decreases and converges to different constant values depending on $n_\alpha$ and $r_c$. The combined fiber loss decreases with an increase in $n_\alpha$ and $r_c$ because it improves optical confinement.

Material loss limit value $L_{mat}$ of the TE$_{0m}$ mode is illustrated in Fig. 11 as a function of core radius $r_c$ and the cladding index.
values. Cladding material losses significantly influence fiber mode, we have the mode as a function of the cladding index contrast. Fiber parameters are the same as those in Fig. 10. The number of cladding pairs is fixed at $N = 40$, which is always larger than $N_{tr}$ for all cases. An extremely large loss at small $r_c$ results from the mode cutoff, as pointed out in Fig. 7. $L_{mat}$ decreases with increases of $n_a$ and $r_c$. The third-power dependence of $L_{mat}$ on $r_c$ can be seen despite $n_a$ here as well as in Fig. 7. For example, for $n_a = 2.5$ and $r_c = 0.2 \mu m$ of the TE$_{01}$ mode, we have $L_{mat} = 7.22 \times 10^{-2} \text{ dB/km}$. For the same $r_c$, we have $L_{mat} = 2.84 \times 10^{-2}$ and $1.57 \times 10^{-2} \text{ dB/km}$ for $n_a = 3.5$ and 4.5, respectively. For a fixed value of $n_a = 2.5$, we obtain $L_{mat} = 4.54 \times 10^{-3}$ and $5.66 \times 10^{-4} \text{ dB/km}$ for $r_c = 5.0$ and 10.0 $\mu m$, respectively. An increase in the core radius effectively reduces the fiber loss.

A plot of transition point $N_{tr}$ of the TE$_{01}$ mode is shown in Fig. 12 as a function of core radius $r_c$ and the cladding index contrast. The fiber parameters are the same as those in Fig. 10. For a small value of $r_c$, $N_{tr}$ increases with increased radial mode number $\mu$. At minimum $r_c$ corresponding to the guiding limit of the individual mode, the $N_{tr}$ of the TE$_{01}$ mode becomes identical despite the mode number $\mu$, although it depends on $n_a$. For instance, $N_{tr} = 21.8$, 13.7, and 10.8 are obtained at $n_a = 2.5$, 3.5, and 4.5, respectively. $N_{tr}$ is reduced with increasing $r_c$ and $n_a$ and converges to constant values with increasing $r_c$ regardless of the mode number, as expected from Fig. 9. For example, the converging values of $N_{tr}$ are 15.6, 10.6, and 8.7 for $n_a = 2.5$, 3.5, and 4.5, respectively.

6. WAVELENGTH DEPENDENCE OF COMBINED FIBER LOSS

Both $L_{mat}$ and $N_{tr}$ show their minimum values for the TE$_{01}$ mode among all modes in the hollow-core Bragg fiber. The TE$_{01}$ mode can be used as a propagation mode in the Bragg fiber because other modes decay faster. The wavelength dependence of the combined fiber loss of the TE$_{01}$ mode is investigated here.

A. Photonic Band Dependence of Combined Fiber Loss

We investigate the wavelength dependence of the combined fiber loss over a wide range of wavelengths. We set $n_a = 2.5$, $n_o = n_{ex} = 1.5$, $r_c = 2.0 \mu m$, and $N = 17$. The fiber parameters are first prescribed to satisfy the QWS condition at $\lambda_{QWS} = 1.0 \mu m$, and then we vary the wavelength by firmly fixing the remaining parameters. In particular, our attention is focused on the performance near $\lambda_{QWS}/t$, where $t$ is an integer.

The PBG dependence of the combined fiber loss of the TE$_{01}$ mode of the hollow-core Bragg fiber is plotted in Fig. 13 for three kinds of $\alpha_a = \alpha_c$. We find three low-loss regions near $\lambda_{QWS}/t$ with odd $t$. It is natural that the minima decreases with decreasing $\alpha_o$. The minimum values decrease with increasing odd $t$. For example, for $\alpha_o = \alpha_a = 1.0 \text{ dB/km}$, the minimum losses are $2.96 \times 10^{-2}$, $1.91 \times 10^{-3}$, and $7.16 \times 10^{-4} \text{ dB/km}$ for $t = 1$, 3, and 5, respectively. Wavelengths $\lambda_{min}$ exhibiting the minimum loss are 0.99, 0.327, and 0.195 $\mu m$ for identical $t$ values. Cladding material losses significantly influence fiber minimum loss, but have little influence on the PBG width. The combined fiber losses for even $t$ hardly depend on cladding material losses because their large fiber losses depend on the CL only. The values of effective index $\beta_R/k_0$ were 0.95237, 0.988, 0.995, 0.997, and 0.998 for $t = 1$ to 5, respectively and, thus, show relatively high values despite different values of $t$. For $t = 1$ estimated from the infinite periodic cladding and QWS case [17], we have the effective index $\beta_{R_{cl}}/k_0 = 0.95238$, indicating an approximate equality between effective indices, namely, propagation constants, described in Appendix A.
The PBG dependence of the combined fiber loss can be explained by considering the in-phase condition at the core-cladding boundary for waves reflected from each cladding interface [21]. If the phase of the reflected waves is in phase (out of phase), then the optical confinement becomes excellent (bad) for an odd (even) \( \ell \) as a consequence of the constructive (destructive) interference. Therefore, a low-loss region is obtained for an odd \( \ell \), but an extremely large loss is obtained for an even \( \ell \). In this case, it is essential that the effective index be nearly unchanged despite \( \ell \), as shown above.

A significant loss difference between odd- and even-numbered PBGs has also been observed in the bend loss of an all-solid silica PBF [22]. The PBG order dependence of bend loss has semiquantitatively been explained by a picture based on the overlap strength between the modes of neighboring rods or by evaluating the critical bend radius [22].

B. Wavelength Dependence of Material Loss Limit Value \( L_{\text{mat}} \)

Material loss limit value \( L_{\text{mat}} \) is shown in Fig. 14 for the TE\(_{01}\) mode as a function of wavelength, core radius \( r_c \), and cladding index contrast. Cladding material losses are prescribed at \( \alpha_a = \alpha_b = 10 \text{ dB/km} \). Only the fundamental PBG is shown here. Fiber parameters are first prescribed to satisfy the QWS condition at \( \lambda_{\text{QWS}} = 1.0 \mu \text{m} \), and then the wavelength is varied by keeping the remaining parameters fixed. \( L_{\text{mat}} \) decreases with increasing \( r_c \) and \( n_a \). For example, for \( n_a = 3.5 \) and \( r_c = 2.0 \mu \text{m} \), the minimum fiber loss of 2.57 \( \times 10^{-3} \text{ dB/km} \) is obtained at \( \lambda_{\text{mat,1}} = 0.920 \mu \text{m} \), which shifts from \( \lambda_{\text{QWS}} \). If \( r_c \) is chosen to be 2.0, 5.0, and 10.0 \( \mu \text{m} \), then we have fiber losses less than \( 10^{-1}, 10^{-2}, \) and \( 10^{-3} \text{ dB/km} \), respectively, for the fundamental PBG. These fiber losses are less than about \( 10^{-2}, 10^{-3}, \) and \( 10^{-4} \) times of cladding material losses, respectively. On the other hand, the PBG width has a stronger dependence on \( n_a \) than on \( r_c \).

7. CONCLUSION

The CL, including cladding material losses, was comprehensively studied for the TE, TM, and hybrid modes of hollow-core Bragg fibers mainly under the QWS condition using the multilayer division method. In particular, detailed numerical data were presented for the TE\(_{01}\) mode. If the core radius is set to 2.0, 5.0, and 10.0 \( \mu \text{m} \) under appropriate fiber design, then we have fiber losses below about \( 10^{-2}, 10^{-3}, \) and \( 10^{-4} \) times of cladding material losses, respectively.

As number \( N \) of the cladding pairs increases, the combined fiber loss of all modes decreases and converges to a constant loss \( L_{\text{mat}} \) peculiar to cladding material losses and the mode. For small \( N \), the combined fiber loss tends to approach a loss determined only by CL \( L_{\text{conf}} \). Material loss limit value \( L_{\text{mat}} \) decreases with increasing core radius and cladding index contrast. \( L_{\text{mat}} \) decreases in inverse proportion to the third and first powers of the core radius for the TE and the other three modes for a sufficiently large core radius. Transition point \( N_{\text{tr}} \) is defined by a crossing point of the asymptotes of \( L_{\text{conf}} \) and \( L_{\text{mat}} \). \( L_{\text{mat}} \) for various cladding material losses can be estimated by those for particular material losses.

Odd-numbered PBGs exhibit a low-fiber loss while even-numbered PBGs exhibit an extremely large fiber loss, if fiber parameters are prescribed to satisfy the QWS condition at the fundamental PBG. Fiber losses at the fundamental PBG decrease with increasing cladding index contrast and core radius. The width of the fundamental PBG strongly depends on the core radius, but hardly on the cladding index contrast.

APPENDIX A: MODE DESIGNATION AND PROPAGATION CONSTANT EVALUATION

For a Bragg fiber with infinite periodic cladding, the real propagation constant \( \beta_{Bn} \) of any mode can be expressed in a simple analytical form under the QWS condition [17]. For a finite number \( N \) of periodic cladding pairs with no material loss, we assume that the propagation constant \( \beta \) be represented by \( \beta_{Bn} + i\beta_{IN} \). Then, the lateral propagation constant \( k_i \) of the \( i \)th layer is treated as complex. The \( \beta_{Bn} \) and \( \beta_{IN} \) are simultaneously obtained by solving the eigenvalue equation in the multilayer division method [12], and \( |\beta_{IN}| \) is markedly lower than \( |\beta_{Bn}| \). The \( \beta_{Bn} \) is close to the \( \beta_{Bn} \) for the same fiber parameters and wavenumber as those of the infinite case, as long as \( N \) is not extremely small. For the present case, where the cladding material loss \( \alpha_i \) is included in terms of the
complex refractive index $\tilde{n}_i$, the real part $\beta_r$ is close to the $\beta_{RN}$ for the same fiber parameters and $N$ as those of the finite $N$ case, as long as the material loss is not so large (see the second paragraph of Subsection 2.B). The imaginary part $\beta_I$ approaches the $\beta_{IN}$ with decreasing $N$, as shown in Fig. 3. It is also possible for us to discriminate between HE and EH modes by using the sign of real part of the $P$ parameter [12] even in the present case. The imaginary part $\beta_I$ of each mode correctly designated is evaluated in this way.

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