Handling Engineering 2 Final Exam.

1. Let us formulate the one-dimensional viscoplastic deformation of a beam of length L and area of cross section A illustrated in Figure 1. Object deformation is described by Maxwell model, where E and η denote Young's modulus and viscous modulus of the object material. Let ρ be the line density of the object. Assume that E, η, ρ, and A are constant. The left end point of the object is fixed to space while force $f(t)$ is applied to the right end point of the object at time t. Let us describe the object deformation by five nodal points: P_0 through P_4. Derive a set of dynamic equations by applying a finite element approach.

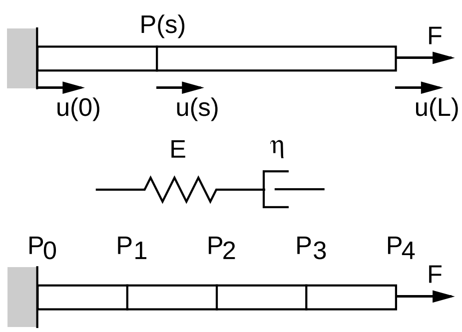

Figure 1: Maxwell object deformation

2. Let us bend a paper of length L and of uniform width on a table by decreasing the distance between two fingers pushing the both end of the paper. Assume that the bend is one-dimensional and investigate the cross section of the paper, as illustrated in Figure 2. Let s be the distance from the left end along the paper. Let $P(s)$ be a point on the paper specified by distance s. Let $\theta(s)$ be the angle from the horizon at point $P(s)$. Assume that bend rigidity R_f of the paper is constant. Let $x(s)$ and $z(s)$ be coordinates at point $P(s)$. Let ℓ be the distance between the two fingers. Assume that the gravitational potential energy is negligible.

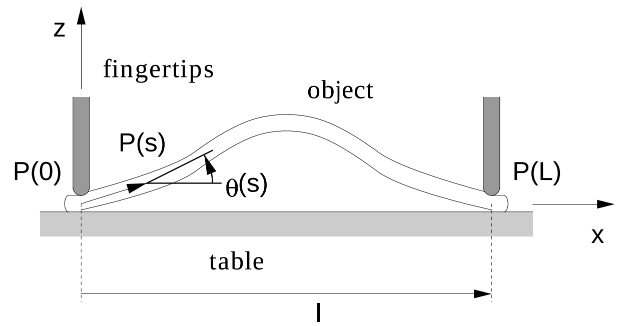

Figure 2: Bend of paper on table

The deformed shape of a paper can be computed by solving the following variational problem:

$$\begin{aligned} \text{minimize} \quad & U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)^2 \mathrm{d}s \\ \text{subject to} \quad & \theta(0) = 0, \quad \theta(L) = 0, \\ & x(L) = \int_0^L \cos\theta(s)\,\mathrm{d}s = \ell, \\ & z(L) = \int_0^L \sin\theta(s)\,\mathrm{d}s = 0. \end{aligned}$$

(a) Convert the above variation problem with equational constraints into another variational problem without any constraints.

(b) Applying the Euler-Lagrange equation in variation, show that the above variation problem is equivalent to the following differential equation:

$$R_f \frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} + \lambda_x \sin\theta - \lambda_y \cos\theta = 0,$$

where λ_x and λ_y are Lagrange multipliers. Note that the above equation is equivalent to the equation of motion of a simple pendulum.