Handling Engineering 2 Report

Answer one of the following questions.

1. Simulate an indirect simultaneous positioning of an extensible 2D object illustrated in Figure 1. Three points marked as circles on the object must be guided to their desired location marked as crosses by controlling the position of three manipulated points marked as triangles, as shown in the figure. You may assume that the control process is static. Introduce the discrepancy between physical parameters in control law and actual ones to discuss the robustness of the control law.

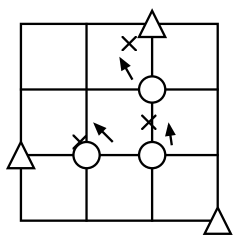

Figure 1: Indirect simultaneous positioning

2. Simulate the 1D viscoplastic deformation of an object. Let E be Young's modulus, η be viscous modulus, and ρ be line density of the object. In addition, simulate the 1D rheological deformation of an object. Let E be Young's modulus, η_1 and η_2 be viscous moduli, and ρ be line density of the object.

3. Let us bend a paper of length L and of uniform width on a table by decreasing the distance between two fingers pushing the both end of the paper. Assume that the bend is one-dimensional and investigate the cross section of the paper, as illustrated in Figure 2. Let s be the distance from the left end along the paper. Let $P(s)$ be a point on the paper specified by distance s. Let $\theta(s)$ be the angle from the horizon at point $P(s)$. Bend potential energy U is then formulated as

$$U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s} \right)^2 \, \mathrm{d}s,$$

where R_f denotes the bend rigidity of the paper. Assume that bend rigidity R_f is constant. Let $x(s)$ and $z(s)$ be coordinates at point $P(s)$, which are described as

$$x(s) = \int_0^s \cos\theta(u) \, \mathrm{d}u,$$
$$z(s) = \int_0^s \sin\theta(u) \, \mathrm{d}u.$$

Let ℓ be the distance between the two fingers. Assume that the gravitational potential energy is negligible.

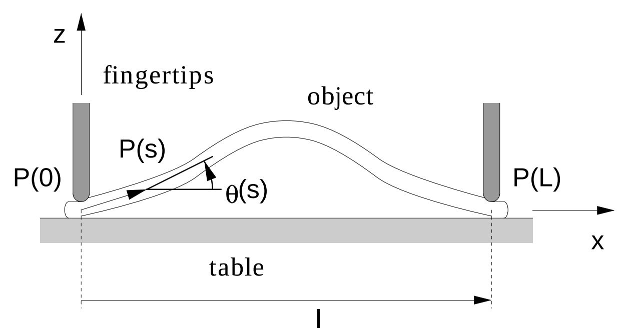

Figure 2: Bend of paper on table

The statically stable deformed shape of a paper can be computed by solving the following variational problem:

$$\begin{aligned} \min \quad & U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s} \right)^2 \, \mathrm{d}s \\ \text{subject to} \quad & \theta(0) = 0, \quad \theta(L) = 0, \\ & x(L) = \int_0^L \cos\theta(s) \, \mathrm{d}s = \ell, \\ & z(L) = \int_0^L \sin\theta(s) \, \mathrm{d}s = 0. \end{aligned}$$

Applying a finite element analysis to the above variational problem, compute the deformed shape of a paper.

4. Let us formulate the dynamic bend deformation of a paper of length L and of uniform width. Assume that the bend is one-dimensional and investigate the cross section of the paper. Let s be the distance from the left end along the paper. Let $P(s)$ be a point on the paper specified by distance s. Let $\theta(s,t)$ be the angle from the horizon at point $P(s)$ at time t. Let D be the weight per unit length of the paper. Assume that the weight per unit length D is constant. Kinetic energy of the paper is then formulated as

$$T = \int_0^L \frac{1}{2} D \left\{ \left(\frac{\partial x}{\partial t} \right)^2 + \left(\frac{\partial y^2}{\partial t} \right)^2 \right\} \mathrm{d}s,$$

where

$$\begin{aligned} x(s,t) &= \int_0^s \cos\theta(u,t) \, \mathrm{d}u, \\ z(s,t) &= \int_0^s \sin\theta(u,t) \, \mathrm{d}u. \end{aligned}$$

Bend potential energy U is formulated as

$$U = \int_0^L \frac{1}{2} R_f \left(\frac{\partial\theta}{\partial s} \right)^2 \mathrm{d}s,$$

where R_f denotes the bend rigidity of the paper. Assume that bend rigidity R_f is constant. Assume that torque τ is applied to the object at the left end point and the right end point is free to move. The dynamic bend deformation of a paper can be computed by solving the following variational problem:

$$\text{V.I.} = \int_{t_0}^{t_1} \{ T - U + \tau\theta(0) \} \, \mathrm{d}t.$$

Applying a finite element analysis to the above variational problem, compute the dynamic deformation of a paper.