

Analytical Mechanics Final Exam.

1. A bead of mass m is constrained to slide along a thin, circular hoop of radius l . The hoop rotates with a constant angular velocity ω in a horizontal plane around point O on its rim, as illustrated in Figure 1.

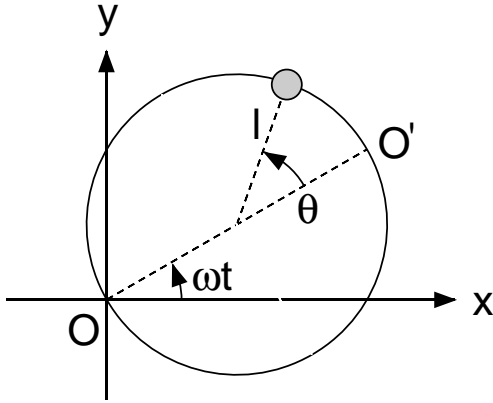


Figure 1: Bead moving along hoop

- Find the Lagrange equation of motion of the bead.
- Let O' be the point on the hoop rim diametrically opposite to point O . Show that the bead oscillates like a simple pendulum about point O' .

2. Let us investigate the transversal vibration of a beam. The beam is of length L and its one end is fixed on a wall, as illustrated in Figure 2. Force $f(t)$ is applied to the other end at time t . Let μ be the line density of the beam, E be its Young's module, and I be its geometrical moment of inertia. Let x be the distance from the wall and $u(x, t)$ be the transversal displacement at distance x and time

t , as illustrated in the figure. Kinetic energy and bend potential energy of the beam are then described as follows, respectively:

$$T = \int_0^L \frac{1}{2} \mu \left(\frac{\partial u}{\partial t} \right)^2 dx,$$

$$U = \int_0^L \frac{1}{2} EI \left(\frac{\partial^2 u}{\partial x^2} \right)^2 dx.$$

Work done by the external force is described as

$$W = f(t)u(L, t).$$

Assuming that μ , E , and I are constant, compute the variation of action integral

$$\delta \int_{t_1}^{t_2} (T - U + W) dt$$

and derive a differential equation that $u(x, t)$ must satisfy.

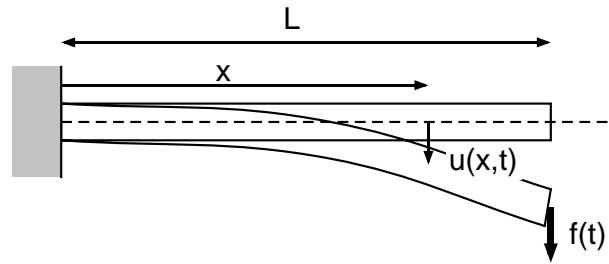


Figure 2: Transversal vibration of beam