Motivation of this Research

The legged robot was expected to be an environment-accessible platform because of its environment adaptability.

The quadruped walking robot will be one of the most practical locomotion machines to move about on uneven terrain and is stablest while walking in static state.

In this study, we limit the discussion on the static walking of a quadrupled robot that is always statically stable, and present an omni-directional walking gait to realize the omni-directional static walking of the robot.
## Studies on Static Walking of Quadrupled Robot

### Static walking gaits

- Crawl gait (McGhee, 1968)
- Crab gait (Hirose et. al, 1986)
- Rotation gait (Hirose et. al, 1986)

### Gait transition between static gaits

- Standing posture transformation gait, gait transition between “standing postures” (Hirose et. al, 1988)
- Semi-autonomous walking gait (Adachi et. al, 1998)
- Gait transition in cooperation of sideways’ motion of torso (Hugel et. al, 1999)
Contents of This Research

Problems of previous gait-transition techniques

- Standing posture transformation gait must have the torso stop at least of 4 steps
- Semi-autonomous walking gait must use the border of the leg work space
- Gait transition in cooperation of sideways’ motion of torso must cooperate with sideways’ motion of torso

Introduce **Omni-directional walking gaits** to perform the omni-directional walking continuously with the least time of torso stop
The Quadruped Robot chooses the crawl gait or rotation gait from the center of turning and transfers the gaits from one to another continuously.

Standard gait planning: After selecting the gait from turning center, plan the leg positions and trajectories with legs’ reachable ranges.

1. Select a gait from the position of turning center.
2. Derive the landing position of the swinging leg and the lifting position of the supporting leg from the legs’ reachable range.
3. Plan the leg trajectories that connect corresponding points.

Successive gait-transition: Transfer the gait successively corresponding to the change of turning center.

1. Select next gait and derive the next-step legs’ positions from new position of turning center.
2. Move the legs from current legs’ positions to new legs’ positions successively.
Design the foots before and after gait transition to have common positions, and plan the leg trajectories to make the 2 supporting legs formed a diagonal line locate at the basic legs’ positions at DTE point.

Perform the gait-transition at one step before DTE point.
Planning of Standard Gaits

Selection of gait pattern: The walking gait pattern is selected from the position of the turning center

\[ Q = \text{Rot}(k, \frac{\pi}{2}) \left[ \begin{array}{c} v_{in} \\ \frac{\theta_{in}}{\dot{\theta}_{in}} \\ -c_{iz} \end{array} \right]^T \]

Derivation of leg’s landing position and leg’s lifting position:
The landing position of the swinging leg and the lifting position of the supporting leg are derived from the rotation speed, the selected gait, and the legs’ reachable ranges

Planning of leg trajectories: The leg trajectory of the swinging leg is a curve connected its start position \( p_{si} \) and its end position \( p_{ei} \) with account of height \( h_{sw} \). The trajectories of the supporting legs are a circle
Selection of Gait Pattern

(a) Left Turn

(1) X-crawl
(2) RX-crawl
(3) Y-crawl
(4) RY-crawl
(5) O-rotation

(b) Right Turn

(2) RX-crawl
(1) X-crawl
(4) RY-crawl
(3) Y-crawl
(6) RO-rotation
**Leg’s Landing Position and Leg’s Lifting Position**

Landing position of the swinging leg:

\[ \phi_{ei} = \phi_{ci} + K_{qi} \theta \]

\[ p_{ei} = Q + \begin{bmatrix} r_i \cos \phi_{ei} \\ r_i \sin \phi_{ei} \\ 0 \end{bmatrix} \]

Lifting position of the supporting leg:

\[ \phi_{hi} = \phi_{ei} - \dot{\theta}(T - T_{sw}) \]

\[ p_{hi} = Q + \begin{bmatrix} r_i \cos \phi_{hi} \\ r_i \sin \phi_{hi} \\ 0 \end{bmatrix} \]

Partition coefficient \( K_{qi} \) is \( K_{qfi} \) or \( K_{qri} \)

<table>
<thead>
<tr>
<th>Leg Position</th>
<th>X-crawl</th>
<th>RX-crawl</th>
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<tbody>
<tr>
<td>( K_{qfi} )</td>
<td>( K_{qri} )</td>
<td>( K_{qfi} )</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
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<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.25</td>
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<table>
<thead>
<tr>
<th>Y-crawl</th>
<th>RX-crawl</th>
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</thead>
<tbody>
<tr>
<td>( K_{qfi} )</td>
<td>( K_{qri} )</td>
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<tr>
<td>1</td>
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<tr>
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<tr>
<td>4</td>
<td>0.25</td>
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<table>
<thead>
<tr>
<th>O(RO)-rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{qfi} )</td>
</tr>
<tr>
<td>1, 2, 3, 4</td>
</tr>
</tbody>
</table>
Planning of Leg Trajectories

Leg trajectory of the swinging leg:

\[
p_{di} = \begin{bmatrix}
p_{six} - p_{eix} \\
p_{siy} - p_{eiy} \\
h_{sw} \{1 - \cos \left( \frac{2\pi}{T_{sw}} t_i \right) \}
\end{bmatrix}
\begin{bmatrix}
\frac{\pi}{T_{sw}} t_i \\
\frac{\pi}{T_{sw}} t_i \\
\frac{\pi}{T_{sw}} t_i 
\end{bmatrix} + \frac{1}{2} (p_{si} + p_{ei}), \quad \dot{p}_{di} = \begin{bmatrix}
-\frac{1}{2} (p_{six} - p_{eix}) \frac{\pi}{T_{sw}} \sin \left( \frac{\pi}{T_{sw}} t_i \right) \\
-\frac{1}{2} (p_{siy} - p_{eiy}) \frac{\pi}{T_{sw}} \sin \left( \frac{\pi}{T_{sw}} t_i \right) \\
h_{sw} \frac{\pi}{T_{sw}} \sin \left( \frac{2\pi}{T_{sw}} t_i \right)
\end{bmatrix}
\]

Leg trajectory of the supporting leg:

\[
p_{di} = Q + \begin{bmatrix}
r_i \cos \phi_{di} \\
r_i \sin \phi_{di} \\
0
\end{bmatrix}, \quad \dot{p}_{di} = \begin{bmatrix}
-r_i \sin(\phi_{di}) \dot{\phi}_{di} \\
r_i \cos(\phi_{di}) \dot{\phi}_{di} \\
0
\end{bmatrix} = \begin{bmatrix}
r_i \sin(\phi_{di}) \dot{\theta} \\
-r_i \cos(\phi_{di}) \dot{\theta} \\
0
\end{bmatrix}
\]

\[
\phi_{di} = \phi_{ei} - \dot{\theta} (t_i - T_{sw})
\]
While the position of new turning center is given, we first select the corresponding gait and derive the corresponding legs’ positions, then move the legs from the current positions to new positions, to perform the successive gait-transition.

- Gait-transition from crawl gait to crawl gait
- Gait-transition from crawl gait to rotation gait
- Gait-transition from rotation gait to crawl gait
- Gait-transition from rotation gait to rotation gait
Gait-transition from Crawl to Crawl

Case 1: Front leg is first swinging leg in walking direction of new gait

Case 2: Back leg is first swinging leg in walking direction of new gait
Gait-transition from Crawl to Rotation

(a) Case 1

(b) Case 2
Gait-transition from Rotation to Crawl

Step 1

Step 2

Step 3

New gait
Gait-transition from Rotation to Rotation

Step 1

Step 2

Step 3

Step 4

New gait
Comparison of each Gait-transition Time

<table>
<thead>
<tr>
<th>→</th>
<th>X</th>
<th>RX</th>
<th>Y</th>
<th>RY</th>
<th>O</th>
<th>RO</th>
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</thead>
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<tr>
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<td>1</td>
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<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>X</td>
<td>$\frac{1}{2}T$</td>
<td>—</td>
<td>—</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{1}{2}T$</td>
<td>$\frac{3}{4}T$</td>
</tr>
<tr>
<td>RX</td>
<td>—</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{1}{2}T$</td>
<td>—</td>
<td>$\frac{1}{2}T$</td>
<td>$\frac{3}{4}T$</td>
</tr>
<tr>
<td>Y</td>
<td>$\frac{1}{2}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{1}{2}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{1}{2}T$</td>
<td>—</td>
</tr>
<tr>
<td>RY</td>
<td>$\frac{1}{2}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{1}{2}T$</td>
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</tr>
<tr>
<td>O</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
</tr>
<tr>
<td>RO</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
<td>$\frac{3}{4}T$</td>
</tr>
</tbody>
</table>

Note: $X, RX, Y, RY, O, RO$ is each gait pattern

A rule “the swinging motion of the leg is omitted if the lifting position and the landing position of the leg are same” was utilized to shorten the gait-transition time.
Simulations (*Crawl to Crawl*)

- **Case 1**
  - (a) Walking path
  - (b) Stability margin and legs’ state

- **Case 2**
  - (a) Walking path
  - (b) Stability margin and legs’ state
Simulations (*Crawl to Rotation*)

- **Case 1**
  - (a) Walking path
  - (b) Stability margin and legs’ state

- **Case 2**
  - (a) Walking path
  - (b) Stability margin and legs’ state
Simulations (*Rotation to Crawl, Rotation to Rotation*)

- **Gait-transition from Rotation gait to Crawl gait**
  
  ![Diagram](image1)
  
  (a) Walking path  
  (b) Stability margin and legs’ state

- **Gait-transition from Rotation gait to Rotation gait**
  
  ![Diagram](image2)
  
  (a) Walking path  
  (b) Stability margin and legs’ state
Experiments

- Walking by each standard gait (forward/backward, left/right, left/right turn, left/right rotation, and straight-line walking with an angle)
- Gait-transition (crawl→crawl, crawl→rotation, rotation→crawl, and rotation→rotation)

- Walking along ‘8’ character
Conclusions

A successive gait-transition method was proposed for a quadruped robot to realize omni-directional static walking. Experimental test was executed to show that

- In the walking experiment of the standard gaits, the robot walked stably

- In the gait-transition experiments, the robot totally walked stably as the simulation

- In the experiment walking along ‘8’ character, the robot walks only by the $X$-crawl gait and its walking direction is changed through the rotation gait, thus is always stable.