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Simulation Algorithm for the Coupling of the Left Ventricular Mechanical Model with Arbitrary Circulation Model

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***Abstract*— The left ventricular mechanical model and the circulation model are important for the cardiovascular dynamics simulation that consider the characteristics of the ventricular cells and the structure of left ventricle (LV). Due to the fact that the cardiovascular dynamics is the result of nonlinear interactions between these two models, simultaneous consideration of both models using a strong coupling method is necessary for an accurate simulation. In this paper, we propose a simulation algorithm that is capable of calculating cardiovascular dynamics model by strong coupling of the left ventricular mechanical model and the circulation model.**

I. INTRODUCTION

The heart is an essential organ since it functions as a pump that drives blood circulation in the body. Understanding the cardiac function is very important not only for the diagnosis of heart diseases but also for the development of a new therapeutic method. Since the heart always operates with the circulation system, simulation of the cardiovascular dynamics is expected to contribute toward further understanding of the cardiac function.

In a cardiovascular dynamics simulation, the blood pressure and the blood flow in LV and the blood vessels are calculated. It is necessary to use a left ventricular model and a circulation model to realize this simulation. Due to the fact that the cardiovascular dynamics is the result of the nonlinear interactions between these two models, simultaneous calculation of both models by a strong coupling method is necessary to obtain an accurate simulation result.

There are two major types of the left ventricular model: the time-varying elastance model[1] and the left ventricular mechanical model[2]. In the time varying elastance model, the pumping action of a ventricle is modelled as a time-varying ventricular capacitance. Though this type of model has the advantage that the coupling calculation with a circulation model is easy to realize, it is difficult to incorporate the characteristics of the ventricular cells and the structure of LV. In the left ventricular mechanical model, the three-dimensional cardiac shape is represented by many elements which correspond to cardiac tissue. This type of model has the advantage that it is easy to incorporate various physiological properties such as myocardial cell orientation, excitation conducting system, mechanical properties, myocardial electrophsyiological model and so on. Therefore, it is very effective to use this type of model to analyze various aspects of the cardiovascular dynamics.

Shape deformation of the left ventricular mechanical model can be calculated by using the finite element method that solves matrix equations. When we consider a simulation model which is based on two or more phenomena, calculation of a single matrix that represents the whole model is often employed to achieve high simulation accuracy. Such method is called matrix coupling method and an accurate analysis can be performed even when the nonlinear interactions exsist in the phenomena under consideration. Since the whole simulation model is represented as a single matrix, however, construction of a new calculation system is required to exchange a simulation model. For this reason, the flexibility of the system is low. On the other hand, the weak coupling method, which uses independent simulators for each phenomenon and calculates them sequentially, has advantages in the system flexibility. However, the accuracy of simulation may become poor in some simulation models.

In this paper, we propose an accurate simulation algorithm for the left ventricular mechanical model which can couple with various circulation models (Fig.1) to achieve both high system flexibility and high simulation accuracy at the same time.

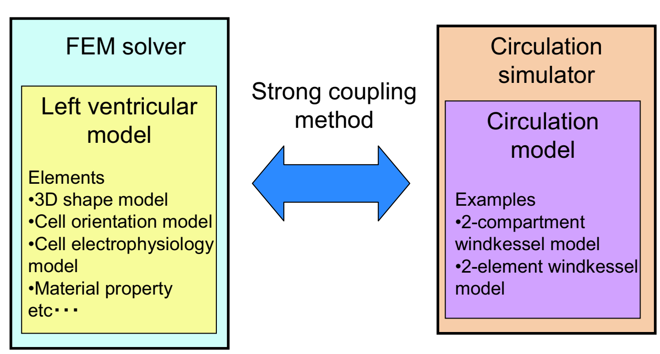

Fig. 1. The goal of our study is to realize strong coupling system under multi-simulator environment.

II. COUPLING SYSTEM

In this study, the cardiovascular dynamics simulation system is constructed by the combination of the left ventricular mechanical model which represents the pressure-volume relation of LV and the circulation model which represents the pressure-volume relation of the circulation system.

A. Elements of the Coupling System

1) left ventricular mechanical model: For the left ventricular mechanical model, the deformation of LV is derived from the relationship between the myocardial cell contraction force, the extension force of the myocardial tissue and the inner pressure of LV. The deformation calculation of LV by using the finite element method is executed by inputting the contraction force and the inner pressure of LV at each time step (Fig.2). This is described as follows

$$V_{lv} = H(P_{lv}, F_b) \tag{1}$$

where H is a function for calculating the transformation and V_{lv}, P_{lv}, and F_b are the left ventricular volume, the inner pressure of LV, and the cell contraction force, respectively.

Note that equation (1) can be calculated in oneway because of the characteristics of the finite element method, i.e. the left ventricular volume can be calculated from the contraction force and the inner pressure of LV, while the contraction force or the inner pressure of LV cannot be derived from the left ventricular volume and the inner pressrue of LV or the contraction force.

Moreover LV has the characteristics that under certain contraction force F_b, the left ventricular volume V_{lv} becomes large when the inner pressure of LV P_{lv} increases. This relation can be illustrated as in Fig.3.

As a cell model, the “Kyoto model”[3] proposed by Noma is used in this study.

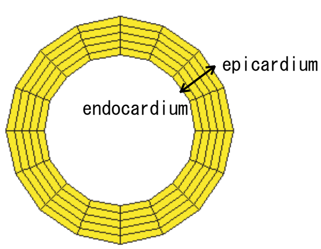

Fig. 2. Left ventricular mechanical model

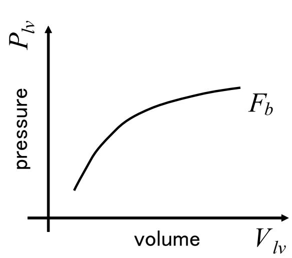

Fig. 3. Pressure-volume relation of the left ventricular mechanical model tells that the pressure becomes large when the volume becomes large under certain contraction force.

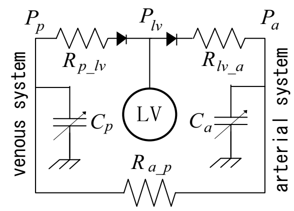

Fig. 4. two compartment windkessel model

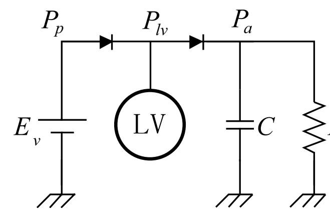

Fig. 5. two element windkessel model

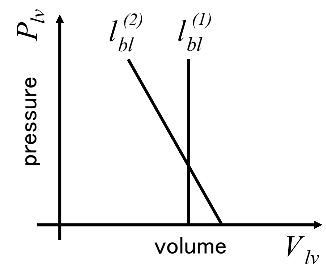

Fig. 6. Pressure-volume relations of the circulation models in the ejection phase.

2) Circulation model: The arterial blood pressure and the blood flow is calculated by the circulation model. The mechanism of the blood vessels is represented by the simple windkessel model which is constructed from the elastance vessel and the resistance vessel[4]. When we pay attention to the method of connecting the windkessel model and the left ventricular mechanical model, there are two types of windkessel models: one is the two compartment windkessel model (Fig.4), and the other is the two element windkessel model (Fig.5). Most of the circulation models can be classified into these two models and the calculation of the cardiovascular dynamics can also be classified into two groups according to the choice of windkessel model.

Let the inner pressure of LV, the arterial blood pressure and the venous blood pressure be denoted as P_{lv}, P_a and P_p, and the left ventricular volume and the vascular volume be represented by V_{lv} and V_a, respectively. The relationship among these parameters in ejection phase can be described as follows.

For the two compartment windkessel model, the blood flow is calculated from the inner pressure of LV P_{lv} and the arterial pressure P_a

$$\frac{\mathrm{d}V_{lv}}{\mathrm{d}t} = -\frac{P_{lv} - P_a}{R_{lv\_a}}. \tag{2}$$

Note that R_{lv_a} indicates the resistance between LV and aorta.

First, when this model is combined with the left ventricular mechanical model, the aortic blood flow is calculated by equation (2). After the left ventricular volume change is determined by the aortic blood flow, the inner pressure of LV is derived from the left ventricular volume and the contraction force in the next time step from the calculation using the finite element method. This tells that the two compartment windkessel model only determines the left ventricular volume, but does not affect the inner pressure of LV. For this reason, pressure-volume relation of the two compartment windkessel model can be illustrated as a line $l_{bl}^{(1)}$ in Fig.6.

For the two element windkessel model, since a resistance does not exist just behind the aortic valve, P_{lv} equals P_a. In this model, the blood flow volume and the arterial blood pressure change are calculated at the same time, thus the blood flow volume itself can not be derived from the difference between the inner pressure of LV and the arterial blood pressure. For this reason, it is necessary to simulate the cardiovascular dynamics by combining the pressure-volume relation of the left ventricular model and the circulation model. This is described as follows

$$\frac{\mathrm{d}V_{lv}}{\mathrm{d}t} = -C \cdot \frac{\mathrm{d}P_a}{\mathrm{d}t} - \frac{P_a}{R} \tag{3}$$

The following equation is derived by discretizing equation (3).

$$C(P_a - P_{a0}) + (V_{lv} - V_{lv0}) + \frac{P_a}{R} \cdot \mathrm{d}t = 0 \tag{4}$$

Equation (4) represents $l_{bl}^{(2)}$ in Fig.6. Here P_{a0} and V_{lv0} correspond to the inner pressure of LV and the left ventricular volume before minute time dt, respectively.

A heart cycle is devided into the following four phases

1) isovolumic contraction phase (if $P_{lv} < P_a$ and $P_{lv} > P_p$)
2) ejection phase (if $P_{lv} \geq P_a$ and $P_{lv} > P_p$)
3) isovolumic relaxation phase (if $P_{lv} < P_a$ and $P_{lv} > P_p$)
4) filling phase (if $P_{lv} < P_a$ and $P_{lv} \leq P_p$)

The aortic valves and the mitral valves are modeled as diodes in both models. For each cardiac phase, the calculation equation of the left ventricular volume change is represented as follows

1) isovolumic contraction phase

$$\frac{\mathrm{d}V_{lv}}{\mathrm{d}t} = 0 \tag{5}$$

2) ejection phase
   a) two compartment windkessel model; equation (2)
   b) two element windkessel model; equation (4)
3) isovolumic relaxation phase

$$\frac{\mathrm{d}V_{lv}}{\mathrm{d}t} = 0 \tag{6}$$

4) filling phase
   a) two compartment windkessel model

$$\frac{\mathrm{d}V_{lv}}{\mathrm{d}t} = \frac{P_p - P_{lv}}{R_{p\_lv}} \tag{7}$$

   b) two element windkessel model

$$P_{lv} = P_p \tag{8}$$

where R_{p_lv} indicates the resistance between vein and LV.

B. System Configuration

For an accurate cardiovascular dynamics simulation that considers interaction between LV and circulation system, coupling simulation of the left ventricular model and the circulation model is necessary. Moreover, new models of the biological systems are yet being developed and continuously improving. Since the accuracy of the current left ventricular mechanical model and the circulation models are not fully sufficient for an accurate cardiovascular dynamics simulation, it is important to evaluate the simulation results calculated by various combinations of newly proposed models.

In this study, a cardiovascular dynamics simulation system is constructed by a general finite element method solver as a simulator of the left ventricular mechanical model and an existing software as a simulator of the circulation model (Fig.7). By using independent software designed with compornent based system architecture, the simulation model can be exchanged easily.

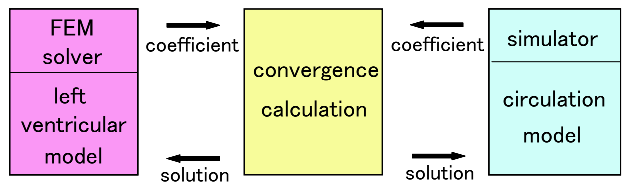

Fig. 7. System configuration

III. COUPLING ALGORITHM

In this study, coupling analysis coresponds to simultaneously obtaining the solution of equation (1) and equation (2) or of equation (1) and equation (4). By using equation (1), V_{lv} can be derived from F_b and P_{lv}, but P_{lv} can not be derived from V_{lv} and F_b. Therefore, to obtain P_{lv}, we use an algorithm which solves the matrix equation by applying a linear approximation method to the circulation model represented by equation (2) or equation (4) and performing convergent calculation at time $t = t_n$. Fig.8 shows the convergent calculation process at time $t = t_n$ in ejection phase of a cardiovascular dynamics simulation using the two element windkessel model.

First, at $t = t_{n-1}$, equation (1) which indicates the pressure-volume relation of the left ventricular model is defined as the curve $F_b^{(t_{n-1})}$, and equation (2) or equation (4) which indicate the pressure-volume relation of circulation model is defined as line $l_{bl}^{(t_{n-1})}$. It can be assumed that the intersection ① is already known. Then pressure and volume at point ① show the inner pressure of LV and the left ventricular volume at $t = t_{n-1}$. The calculation procedure which obtain the solution of the simultaneous equation at $t = t_n$ is shown below.

1) Assume the inner pressure of LV at $t = t_{n-1}$ to be an initial value of the provided inner pressure of LV $P_{lv1}^{(t_n)}$ (point ①).
2) Calculate left ventricular volume $V_{lv2}^{(t_n)}$ from the cell contraction force $F_b^{(t_n)}$ and the inner pressure of LV $P_{lv1}^{(t_n)}$ at $t = t_n$ by using finite element method(point②)

3) For obtaining pressure-volume relation of LV by applying linear approximation method, add minute value to transmural pressure$P_{lv3}^{(t_n)} = P_{lv1}^{(t_n)} + \mathrm{d}P$
4) Calculate left ventricular volume $V_{lv3}^{(t_n)}$ from cell contraction force $F_b^{(t_n)}$ and transmural pressure $P_{lv3}^{(t_n)}$ at $t = t_n$ by using finite element method(point③)
5) Calculate the intersection④($V_{lv4}^{(t_n)}$, $P_{lv4}^{(t_n)}$) of straight line that passes point ② and ③ and the pressure-volume relation of the circulation model $l_{bl}^{(t_n)}$
6) Calculate the left ventricular volume $V_{lv5}^{(t_n)}$ from cell contraction force $F_b^{(t_n)}$ and transmural pressure $P_{lv4}^{(t_n)}$ at $t = t_n$ by using finite element method(point⑤)
7) End the convergent calculation if point④ is very close to point⑤ and proceeds to the next step. Otherwise return to 2) after redefining $P_{lv1}^{(t_n)} = P_{lv5}^{(t_n)}$

For the two element windkessel model, equation (4) is transformed into $V_{lv} = V_{lv0}$ by assuming the compliance C as $C = 1/\infty$ and a resistance vessel R as $R = \infty$. This equation represents the state that the aortic valve is closed. In other words, by deciding the cardiac phase of the heart cycle and by setting the value of compliance and resistance to fit to each cardiac phase, this simulation algorithm can be applied to all cardiac phases. For two compartment windkessel model, because the left ventricular volume of the next step is decided by equation (2), pressure-volume relation of the two compartment windkessel model can be assumed to be close to pressure-volume relation of the two element windkessel model in isovolumic phase. Therefore, this coupling algorithm can be applied to both of the two circulation models. Since most of the circulation models are expressed by the equation similar to either of these two models, the proposed coupling algorithm is generally applicable, and the model can be easily exchanged. Thus, a cardiovascular dynamics simulation system that is flexible in chosing model and is accurate in calculation is realized.

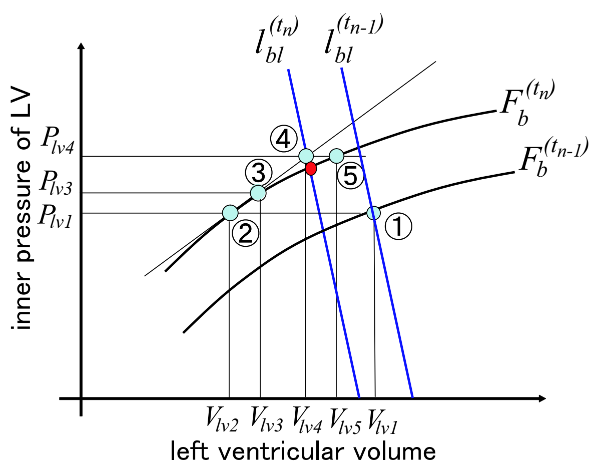

Fig. 8. Coupling algorythm in ejection phase

IV. SAMPLE EXPERIMENTS

In order to verify the effectiveness of the coupling system, cardiovascular dynamics was simulated by using the two compartment windkessel model and the two element windkessel model as a circulation model. The results are shown in Fig.9 and Fig.10, respectively. Each figure shows the pressure-volume relation, when the contraction force is changed in the cell model for each of the circulation models. Fig.9 and Fig.10 verify that the cardiovascular dynamics simulation can be done by using the same simulation algorithm for the different circulation models. Moreover it shows that the change of contraction force in a cell model affects the cardiovascular dynamics.

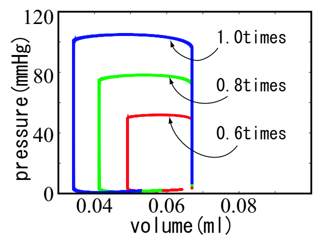

Fig. 9. two compartment windkessel model

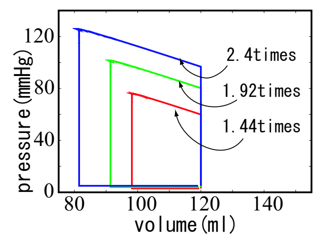

Fig. 10. two element windkessel model

V. CONCLUSION

In this study, we proposed a coupling algorithm which enables cardiovascular dynamics simulation by using the finite element method in a system that consists of independent simulators for the left ventricular mechanical model and a circulation model. By constructing a simulation system from independent software, the simulation models can be easily exchanged. Moreover, we evaluated model changeability of the system and confirmed the generality of our coupling algorithm by simulating cardiovascular dynamics with two circulation models .

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