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Synchronization and its Dynamic Recombination in Associative Memory based on a Chaotic Neural Network

Tomomasa Nagashima(*), Jun Miyazaki, Yoshitaka Shiroki and Isao Tokuda
Department of Computer Science and Systems Engineering,
Muroran Institute of Technology
27-1, Mizumoto, Muroran, Hokkaido 050, JAPAN
(*)E-mail: nagasima@oyna.cc.muroran-it.ac.jp

Abstract. In contrast to standard associative memory dynamics in the class of traditional neural networks with mutual couplings, chaotic neural networks have much variety of dynamics. We investigate retrieval dynamics of associative memory in a chaotic neural network, in which we have stored a set of orthogonal memory patterns. Embedding a set of memory patterns induces specific symmetries to the neural network. By computer experiments, we have found new characteristics in its retrieval dynamics, where at some time duration, our network retrieves in a synchronous manner one of stored pattern and then switches spontaneously to another stored pattern executing a *dynamic recombination of clustering units*. This process seems to continue evermore and every stored pattern is linked dynamically with each other. We discuss on a possible relation between the symmetry property of our network and clustered motions representing stored patterns, and also on a possible mechanism leading to the onset of recombination of clustered pattern.

Keywords: dynamic recombination of memory; synchronization of chaos; chaotic neural network; associative memory; symmetry of dynamic systems

1.Introduction

Artificial Neural Networks have been studied with an aim at developing the tools having flexible information processing abilities such as pattern recognition [3], [19], [26].

(*) To whom all correspondence should be addressed.

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If we focus our attention to their dynamic properties, it would be possible to classify them into deterministic or stochastic networks.

It is well known that the progresses in information processing using deterministic artificial neural networks have been made, for instance, solving combinatorial optimization problems [13], and recalling associative memories [12], [20]. On the other hand, it is also known that their success may be considered as being limited in the sense that they are fundamentally based on simple dynamics of networks, especially the dynamics approaching to some stationary or fixed point solution of the corresponding systems.

In contrast to traditional neural networks which rely on simple deterministic dynamics, chaotic neural networks proposed recently have apparently much variety of dynamics [2], [14], [23] and suggest a new possibility of the information processing based on nonlinear dynamics of neural networks, which attract many researchers [1], [21], [23], [30], [32].

However, almost every interesting dynamics demonstrated in chaotic neural networks for instance, dynamic(chaotic) associative memory [1], [21], chaotic itenerancy [16], [31], [32], etc., seems to relate global properties of their corresponding dynamic systems and therefore, present status for understanding such systems remains far from our satisfaction, because they form difficulties to analyze nonlinear problems with massively coupled elements. We believe at present that it is quite important to classify typical chaotic phenomena in chaotic neural networks which may serve as a basis for designing new tools of information processing.

In this paper, we investigate dynamic retrieval characteristics of a chaotic neural network designed for auto-associative memory. With an intention to clarify essential dynamic characteristics of associative memory, we have stored a set of orthogonal memory patterns in the network.

In the following, we describe our chaotic neural network model and give some discussions on symmetry properties induced by storing orthogonal memory patterns in Section 2. In Section 3, we present the results which have been found in our computer experiments. In this section, we present a new dynamic property in our network designed for associative memory, i.e., clustered dynamics of memories and dynamic recombinations of clustering units, by which every stored pattern is retrieved. We also give a possible mechanism for the onset of dynamic recombinations of clustering units. In Section 4, we summarize our results and give some discussions relating to our paper.

2. The model of chaotic neural network designed for associative memory

2.1. Related recent studies on associative memory

In contrast to the method of the addressed memory employed in conventional computers, the way of reading-out and writing of memory in human beings and animals appears quite different, i.e., it would be based on content addressable memory (CAM) or associative memory. In this regard, realization of associative memory has become one of the most important technological target in artificial neural networks [12], [19]. Recently, encouraged by experimental results on animal brains [10], [29] which suggested crucial roles of nonlinear dynamics in brains, there have been many investigations on artificial neural networks designed for associative memory which are based on complicated nonlinear dynamics [1], [15], [23], [32], [33] beyond traditional dynamics, i.e., simple relaxation to a stable stationary point [13].

While the aims of those investigations looks to be diverse, there seems at least two different directions. One direction has a focus on engineering applications [15],

[23], [33] and is to enhance the associative capabilities in a traditional sense which can be evaluated, for instance, by the static correspondence between input patterns and memory patterns stored in the system.

Among those, Ishii [15] reported an enhancement of the capacity of auto-associative memory based on the investigations on nonequilibrium dynamics of the neural networks.

By extending the globally coupled map (GCM) by Kaneko [16], he devised the neural networks that are capable of associating a stored memory, where the introduction of parameter variations into (symmetric) GCM through an annealing-like mechanism plays an important role.

While GCM reveals a variety of very interesting complicated dynamical phenomena such as frozen cluster attractors, chaotic switching phenomena(chaotic itenerancy) and so on, it is necessary, in general, to supply or devise some mechanisms to original GCM if one wishes to cope with a specific task of information processing, e.g. associative memory as Ishii has considered.

Inspired by the experimental results found in animal brains [10], [29], another stream approach the theoretical models which may cope with complicated neural dynamics in animal brains. In the context of understanding of brain functions, Tsuda [31] stressed an importance of the notion of chaotic itenerancy of memories in human beings, where the memories stored in brains are not static, but they are dynamic and evolves into another memories. While chaotic itenerancy and phenomena similar to it have been reported in various fields nowadays, it seems to lack sound dynamical bases which enables us to understand and utilize those complicated nonlinear phenomena. We believe that in order to develop novel devices which are based on complicated nonlinear dynamics mentioned thus far, it is crucial to improve our understanding which contributes to the fundamentals in designing novel devices.

This motivates our study decried hereafter.

Lastly, showing a strong contrast to every study employing (neural) networks, it should be remarked that there has been published interesting papers which deal with associative memory using 1-D maps [4], [8]. Dimitriev et al devised piecewise linear 1-D maps whose stable cycles are one to one correspondence with specified segments of symbols [8].

They also applied their method in storing and retrieving 2-D patterns and demonstrated very interesting characteristics including intermittent switching of stored patterns. As their method is completely different from any method relying on the abilities of parallel processing mechanisms, it should be an interesting future work to make comparisons between their method and the method based on neural networks.

2.2. Chaotic neural network

The chaotic neural network model we adopt in this paper has been introduced by Aihara et al [2] and is described by the following set of eqs. (1-3):

$$\eta_i(t+1) = k_m \eta_i(t) + \sum_{j=1}^{N} W_{ij} x_j(t) \qquad (1)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \qquad (2)$$

$$x_i(t+1) = f[\eta_i(t+1) + \zeta_i(t+1)] \qquad (3)$$

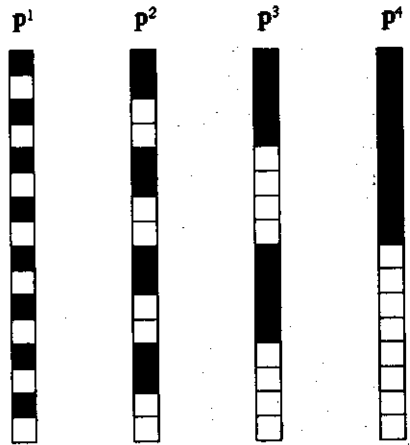


Fig. 1 The set of stored patterns ; The patterns $\mathbf{p}^1$, $\mathbf{p}^2$, $\mathbf{p}^3$, $\mathbf{p}^4$ are orthogonal with each other. A cell with black (white) in each pattern $\mathbf{p}^i$ corresponds to p^k_i=1(0).

where N stands for the number of neurons of the network, $\eta_i(t)$ and $\zeta_i(t)$ are the internal states of i-th neuron at time t; x_i is the output of i-th neuron which is obtained through the sigmoidal function f(•) in eq. (3); W_{ij} represents the coupling between j-th and i-th neurons; k_m, k_r are damping constants; α is the refractory parameter, and a_i is a parameter proportional to the external stimulus to i-th neuron and the threshold value of i-th neuron.

This model is derived based on the followings: (1) an extension of Nagumo-Sato's formal neuron model with refractory effect [22] to the analog neuron model, and (2) the composition of a network by coupling a set of the analog neurons.

It may be worth to note that the internal state of each neuron is determined by 2 variables and it is reduced to single variable if the parameters k_m and k_r are coincide. In that case, the model is simplified to the system of neuronal units with single degree of freedom and reduce to the model which is equivalent to the model obtainable by discretizing (Euler-difference scheme) the Hopfield-Tank's continuos-time neural network [13], [25], [30].

2.3. Associative memory

In order to apply this model to the auto-associative memory, we assume the following coupling between i-th and j-th neurons:

$$W_{ij} = \frac{1}{M}\sum_{k=1}^{M}(2p_i^k - 1)(2p_j^k - 1) \qquad (4)$$

where $\mathbf{p} = (p_1,\ldots,p_N)$ denotes a vector memory pattern stored in the network and $\mathbf{p}^k$ (k=1,...,M) represents k-th vector.

The set of memory patterns (M=4, N=16) adopted in this paper is shown in Fig. 1, and they form orthogonal patterns with each other.

The coupling between neurons by (4) induces specific symmetry properties to the neural network, some of which will be discussed in section 2.4.

2.4. Arguments derived from symmetry property of the neural network

The explicit form of the coupling between neurons reads as follows:

$$4W = \begin{pmatrix}
4 & 2 & 2 & 0 & 2 & 0 & 0 & -2 & 2 & 0 & 0 & -2 & 0 & -2 & -2 & -4 \\
2 & 4 & 0 & 2 & 0 & 2 & -2 & 0 & 0 & 2 & -2 & 0 & -2 & 0 & -4 & -2 \\
2 & 0 & 4 & 2 & 0 & -2 & 2 & 0 & 0 & -2 & 2 & 0 & -2 & -4 & 0 & -2 \\
0 & 2 & 2 & 4 & -2 & 0 & 0 & 2 & -2 & 0 & 0 & 2 & -4 & -2 & -2 & 0 \\
2 & 0 & 0 & -2 & 4 & 2 & 2 & 0 & 0 & -2 & -2 & -4 & 2 & 0 & 0 & -2 \\
0 & 2 & -2 & 0 & 2 & 4 & 0 & 2 & -2 & 0 & -4 & -2 & 0 & 2 & -2 & 0 \\
0 & -2 & 2 & 0 & 2 & 0 & 4 & 2 & -2 & -4 & 0 & -2 & 0 & -2 & 2 & 0 \\
-2 & 0 & 0 & 2 & 0 & 2 & 2 & 4 & -4 & -2 & -2 & 0 & -2 & 0 & 0 & 2 \\
2 & 0 & 0 & -2 & 0 & -2 & -2 & -4 & 4 & 2 & 2 & 0 & 2 & 0 & 0 & -2 \\
0 & 2 & -2 & 0 & -2 & 0 & -4 & -2 & 2 & 4 & 0 & 2 & 0 & 2 & -2 & 0 \\
0 & -2 & 2 & 0 & -2 & -4 & 0 & -2 & 2 & 0 & 4 & 2 & 0 & -2 & 2 & 0 \\
-2 & 0 & 0 & 2 & -4 & -2 & -2 & 0 & 0 & 2 & 2 & 4 & -2 & 0 & 0 & 2 \\
0 & -2 & -2 & -4 & 2 & 0 & 0 & -2 & 2 & 0 & 0 & -2 & 4 & 2 & 2 & 0 \\
-2 & 0 & -4 & -2 & 0 & 2 & -2 & 0 & 0 & 2 & -2 & 0 & 2 & 4 & 0 & 2 \\
-2 & -4 & 0 & -2 & 0 & -2 & 2 & 0 & 0 & -2 & 2 & 0 & 2 & 0 & 4 & 2 \\
-4 & -2 & -2 & 0 & -2 & 0 & 0 & 2 & -2 & 0 & 0 & 2 & 0 & 2 & 2 & 4
\end{pmatrix} \quad (5)$$

From the matrix represented in (5), it is easy to show that there are at least 4 kinds of solutions representing 2-clustered states of the neural network, where every neuronal unit belonging to each cluster (group) behaves identically. To make the statement more precise, let's take one example as follows.

Imagine the third stored pattern ($\mathbf{p}^3$) in Fig.1. By specifying the firing units (F) and non-firing units (N) respectively, this pattern is represented as a vector $\mathbf{p}=(p_1,p_2,\ldots,p_{16})$ where $p_i=1$ ($i \in F$) and $p_i=0$ ($i \in N$).

Note that $F = \{1,2,3,4,9,10,11,12\}$ and $N=\{5,6,7,8,13,14,15,16\}$ for $\mathbf{p}^3$.

Consider the eqs.(1)-(3) of our dynamic system with the coupling (5) and assume that the internal states of each neuron in the network satisfy the conditions $\eta_i(t)=\eta^1$, $\zeta_i(t)=\zeta^1$ ($i\in F$) and $\eta_i(t)=\eta^2$, $\zeta_i(t)=\zeta^2$ ($i\in N$) at time t, where η^i and ζ^i (i=1,2) are the constants. Then, we can show that $\eta_i(t+1)=\tilde{\eta}^1$, $\zeta_i(t+1)=\tilde{\zeta}^1$ ($i\in F$) and $\eta_i(t+1)=\tilde{\eta}^2$, $\zeta_i(t+1)=\tilde{\zeta}^2$ ($i\in N$) hold at time t+1, where $\tilde{\eta}^i$ and $\tilde{\zeta}^i$ (i=1,2) are other constants, because of the relations:

1) $i\in F$

$$\sum_{j\in F} W_{ij} = 2 \text{ , and } \sum_{j\in N} W_{ij} = -2$$

2) $i\in N$

$$\sum_{j\in F} W_{ij} = -2 \text{ , and } \sum_{j\in N} W_{ij} = 2 \qquad (6)$$

This implies a conservation of the 2-clustered motions, in which every neuron in each cluster behaves in a synchronous manner, i.e., behaves identically. By following the same argument, it can be shown that there are at least 4 kinds of 2-clustered motions which are in accord with the set of stored patterns ($\mathbf{p}^1,\mathbf{p}^2,\mathbf{p}^3,\mathbf{p}^4$) in the network.

The special relations contained in the coupling matrix such as:

$$\sum_{j \in F} W_{ij} = 2 \ (i \in F)$$

described in eqs.(6) reflect symmetry properties of our network.

Some relationships between symmetries property of the neural network and clustered (synchronized) solutions which will be necessary for the retrieval dynamics of our network may be summarized as follows.

Let γ be a permutation which represents a transformation of the coordinate system of the dynamic system and Q be the matrix that represents permutation γ. Consider a dynamic system **X**t+1=**FX**t). We say that the dynamic system is symmetric, if **F** commutes with Q, i.e.:

$$\mathbf{F} \circ Q = Q \circ \mathbf{F} \qquad (7)$$

where ° denotes the composition of **F** and Q. Then, it is argued [6], [7] that in the dynamic system with symmetry γ, there should be a solution which satisfies

$$Q\mathbf{X}(t) = \mathbf{X}(t) \quad \text{for all t} \qquad (8)$$

This relation implies an invariance of the solution **X**(t) under the permutation γ. Although it is not trivial to determine how many permutations are there which satisfy symmetry requirement (7) for the neural network of eqs.(1)-(3) with the coupling (5), we can show that at least the following set of permutations satisfy the symmetry requirement (7):

$$\gamma_1\ (z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8,z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16}) = (z_3,z_4,z_1,z_2,z_7,z_8,z_5,z_6,z_{11},z_{12},z_9,z_{10},z_{15},z_{16},z_{13},z_{14})$$

$$\gamma_2\ (z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8,z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16}) = (z_2,z_1,z_4,z_3,z_6,z_5,z_8,z_7,z_{10},z_9,z_{12},z_{11},z_{14},z_{13},z_{16},z_{15}) \qquad (9)$$

$$\gamma_3\ (z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8,z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16}) = (z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16},z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8)$$

and

$$\gamma_4\ (z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8,z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16}) = (z_5,z_6,z_7,z_8,z_1,z_2,z_3,z_4,z_{13},z_{14},z_{15},z_{16},z_9,z_{10},z_{11},z_{12})$$

where $z_i = \{\eta_i, \zeta_i\}$.

Based on this set of permutations which preserve symmetry property of the network, it is shown that each 2-clustered motion which corresponds to one of the stored patterns $\mathbf{p}^k$ (k=1,2,3,4) in the network can be specified by selecting a specific combination of three permutations within γ_i (i=1,2,3,4). Let's take the case of stored pattern $\mathbf{p}^3$.

If we choose γ_1, γ_2, γ_3, then we can show that the 2-clustered motion which corresponds to $\mathbf{p}^3$ is representable as follows. If γ_1 is satisfied in the dynamic system, then there is a solution of the dynamic system satisfying $y_1=y_3$, $y_2=y_4$, $y_5=y_7$, $y_6=y_8$, $y_9=y_{11}$, $y_{10}=y_{12}$, $y_{13}=y_{15}$, $y_{14}=y_{16}$, because of the symmetry relation for the solution $\mathbf{z}(t)=\{z_i(t)\}$ (i=1,2,...,16) which is described in (8). Note that if $z_i=z_j$, then $y_i=y_j$ where $y_i=\eta_i+\zeta_i$. If γ_1, γ_2, γ_3 hold simultaneously, then we have a solution satisfying $y_1=y_2=y_3=y_4=y_9=y_{10}=y_{11}=y_{12}$

and

$y_5=y_6=y_7=y_8=y_{13}=y_{14}=y_{15}=y_{16}$
separately, because of $y_1=y_2$, $y_3=y_4$, $y_5=y_6$, $y_7=y_8$, $y_9=y_{10}$, $y_{11}=y_{12}$, $y_{13}=y_{14}$, $y_{15}=y_{16}$ from γ_2 and $y_1=y_9$, $y_2=y_{10}$, $y_3=y_{11}$, $y_4=y_{12}$, $y_5=y_{13}$, $y_6=y_{14}$, $y_7=y_{15}$, $y_8=y_{16}$ from γ_3. Thus, the solution which satisfies all symmetries required by 3 permutations γ_1, γ_2, and γ_3 leads us to the desired 2-clustered motion which corresponds to $\mathbf{p}^3$.

If it is chosen the set of γ_1, γ_3, γ_4 for the clustered motion corresponding to $\mathbf{p}^1$, the set of γ_2, γ_3, γ_4 for the clustered motion corresponding to $\mathbf{p}^2$, and the set of γ_1, γ_2, γ_4 for the clustered motion corresponding to $\mathbf{p}^4$, we can follow the above statement on the existence of 4-kinds of 2-clustered solutions. In appendix, we have given the explicit form of 4 matrices Q_i which represent the permutations γ_i (i=1,2,3,4).

It should be noted that what we have shown above is merely the existence of 2-clustered (synchronized) solutions in our dynamic system (1)-(3) with the coupling (5) and any stability argument of those solutions is not included.

3. Experimental results

Keeping in mind that there are at least 4 kinds of 2-clustered motions in our network, each of which corresponds to one of stored patterns, we have investigated in detail numerically on the characteristics of retrieval dynamics of our network. Throughout the computer experiment, we take N=16 and fix all parameters except a_i in the eqs. (1)-(3) i.e., $k_m=0.3$, $k_r=0.95$, $\alpha=1.6$, and the steepness parameter ε in the sigmoidal function f(.) as $\varepsilon=0.015$. It is assumed that a_i (i=1,...,N) take an identical value a and the parameter a is treated as the bifurcation parameter.

3.1. One parameter bifurcation

3.1.1. Rough sketch of one parameter bifurcation

To get some insights into the nature of the retrieval dynamics in our network, we studied one parameter bifurcation. Fig. 1 is a bifurcation diagram obtained in the region $a\in[-0.5, 2.0]$ and has been made in the following manner. Choose a starting value of the bifurcation parameter, take a randomly chosen initial condition to the system, and iterate the map described in the eqs.(1)-(3) for a certain interval. Plot their coordinate data against a, then increase the parameter a, and use the last values of coordinate data obtained previously as the initial condition to new system. In this way, Fig. 2 is obtained by plotting the internal state $y_i=\eta_i+\zeta_i$ for i=2. From the bifurcation diagram in Fig. 2, we understand that our system have a rich variety of dynamics.

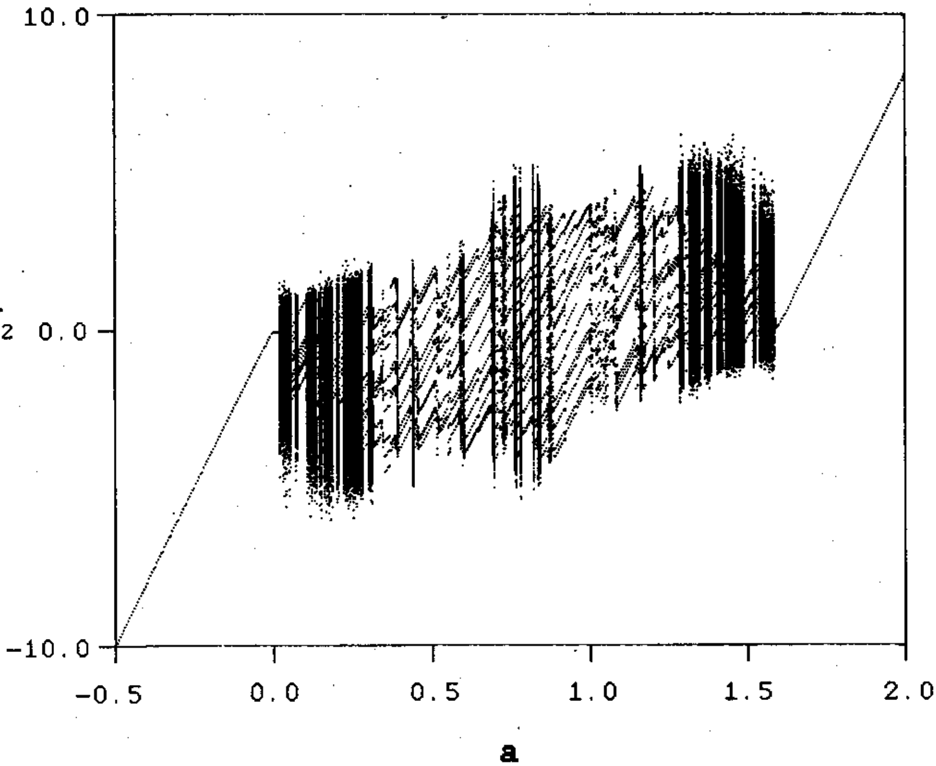


Fig. 2. The bifurcation diagram: The internal state of the second neuron ($y_2 = \eta_2+\zeta_2$) is plotted against bifurcation parameter a ($-0.5 \leq a \leq 2.0$).

While it looks too complicated to analyze them in a systematic manner, we can get a rough idea for the genesis of chaos from the bifurcation diagram, because it reveals a characteristic bifurcation structure in chaotic dynamic systems, i.e., there are parameter regions for chaotic behaviors sandwiched by periodic windows.

3.1.2. Global bifurcation leading to chaos

In order to have more precise information on the bifurcation structure, it is necessary to close up some narrow parameter regions which contain chaotic behaviors.

The following global bifurcation scenario to chaotic behaviors has been obtained by the detailed investigations on narrow chaotic parameter regions inside the interval $a \in [-0.5, 2.0]$. Fig. 3 is an example of the bifurcation diagram obtained by closing up the interval $a \in [0.683, 0.691]$. As is shown in Fig. 3, there is a periodic attractor (with period 14). By increasing parameter a, the periodic attractor bifurcates into another periodic attractor whose period becomes twice of the previous one.

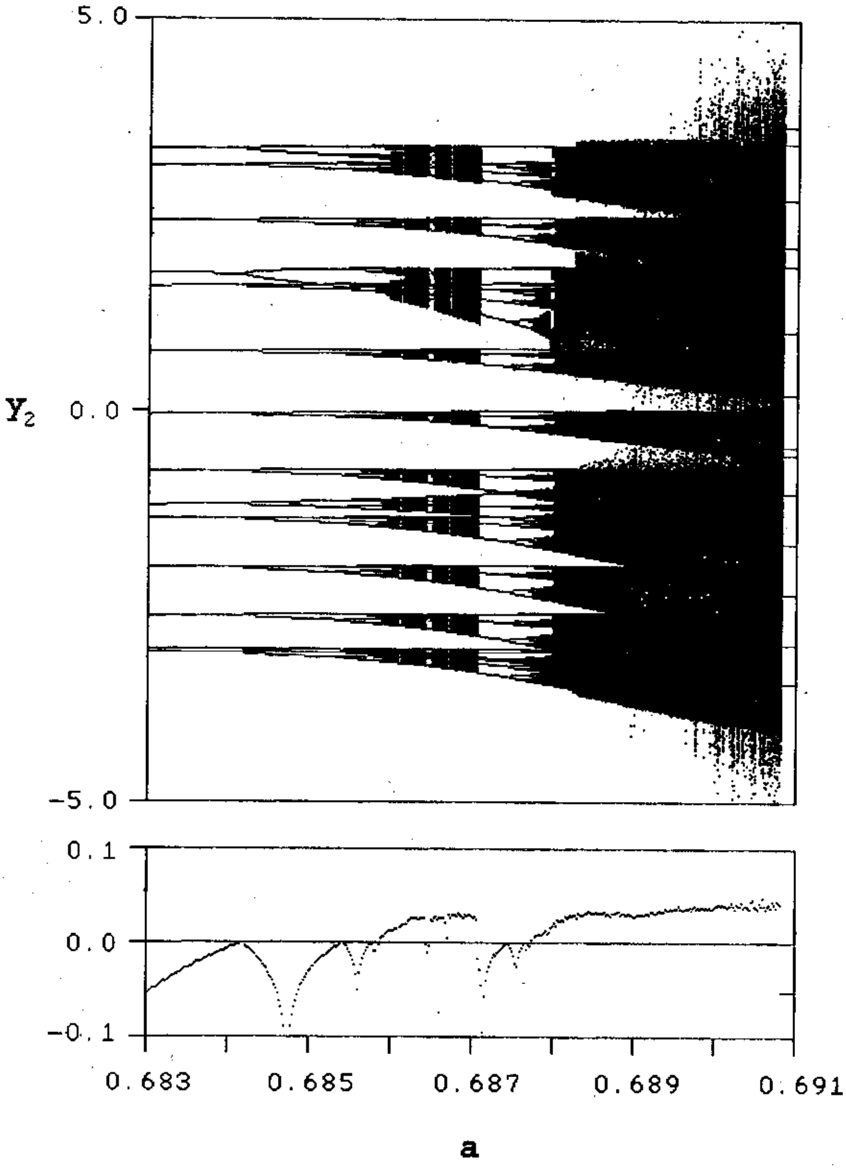


Fig. 3. The enlarged bifurcation diagram and the maximum Lyapunov exponent. The upper figure is an enlargement of Fig. 2 for parameter a (0.683≤ a≤ 0.691). The lower figure gives the maximum Lyapunov exponent vs. a. In chaotic regions, the sign of the maximum Lyapunov exponent takes positive values.

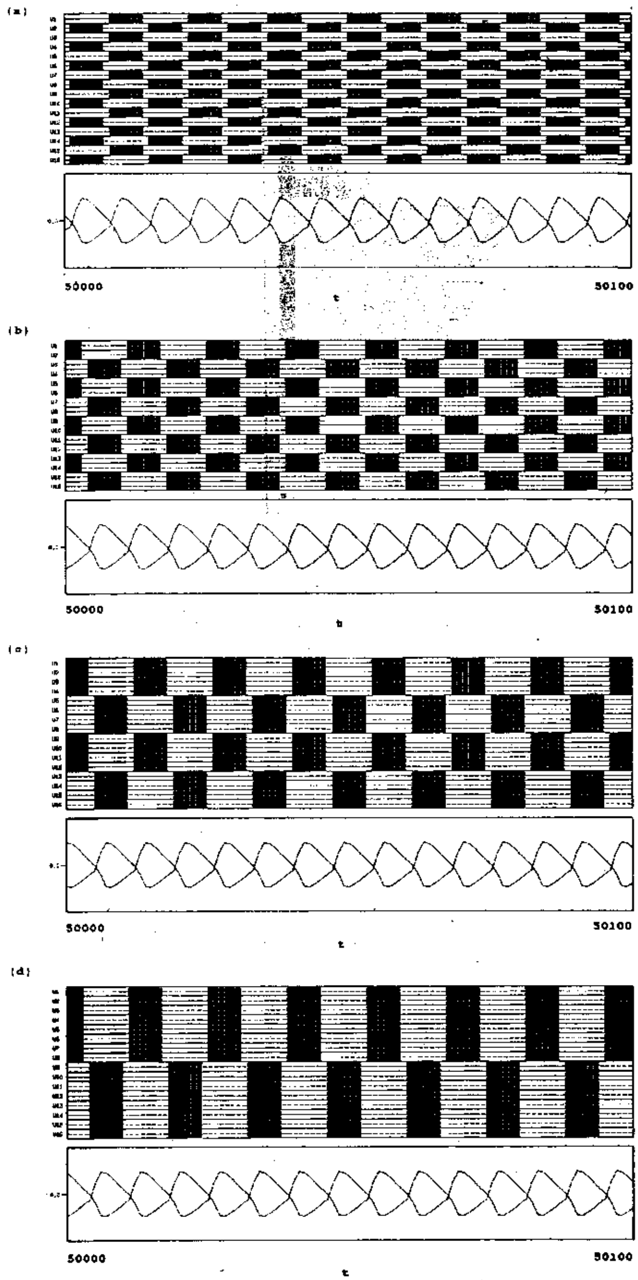


Fig. 4. Examples of typical pattern recalling in periodic region (a=0.683). The network recalls each stored pattern depending on different initial conditions. In each figure (a)-(d), the upper figure shows a temporal development of the output of all neurons, and the lower figure shows that of internal states of all neurons. Note that in the lower figure we have only two curves representing two oscillatory cluster, but this is due to synchronization between neuronal states.

This period doubling bifurcation is repeated and some chaotic attractors seems to appear above the periodic region via the Feigenbaum scenario. Experimentally, it has been confirmed that the period doubling bifurcation continues at least up to 2 to the 5-th power. In order to judge experimentally if the complicated behavior is due to chaos or not, the maximum Lyapunov exponent [28] is evaluated and the result is shown in the lower part of Fig. 3. It should be noted that the bifurcation diagram of Fig. 3 is obtained by setting the initial condition for the starting bifurcation parameter value around the set of the coordinates corresponding to one of the stored patterns $\mathbf{p}^k$ (k=1,2,3,4). For instance, in case of the pattern $\mathbf{p}^4$, the coordinate of the pattern is regarded as x_i=1.0 (i=1,...,8) and x_i=0.0 (i= 9,...,16). We claim that if we choose another class of initial conditions, it may appear different bifurcation diagrams, because we do not know exactly how many attractors are there in the system..

3.2. Synchronized dynamics of stored patterns (Group Synchronization and Quasi-Synchronization)

Even if, from the study of one parameter bifurcation described above, one knows that chaotic behaviors would exist in the network at some parameter value, it is not easy to imagine what kind of dynamics is taking place in the network as a whole. Especially, interrelational information such as phase differences between different neurons is not acquired. One way to get such an interrelational dynamic information among neurons is to observe the temporal development of all internal states simultaneously.

We present in Fig. 4 typical retrieval dynamics in the periodic parameter region given in Fig. 3. In Fig. 4, we display the temporal development of the internal state y_i (in the lower part) and the corresponding output strength x_i (in the upper part) for all i respectively. The figure for the output is made as follows: after quantizing the analogue value of the output strength, the quantized strength is represented by a shaded area in the square assigned for every neuron at each instance. As is demonstrated in Fig. 4, the most characteristic feature of the retrieval dynamics is the formation of dynamic clustering i.e., synchronization, where every neuron in the network oscillates keeping a specific phase relations between neurons.

In other words, the oscillations of all neurons form 2 clusters, i.e., while every neuron in each cluster behaves identically as if single oscillation, the 2 clusters behave keeping their phase difference constant, i.e., half a period. Furthermore, we claim that one of 2-clusterd periodic motions represented in (a)-(d) of Fig. 4 corresponds to one of stored patterns respectively.

It should be mentioned that the dynamic clustering of this type(anti-phase synchronization) continue to exist stably even if the period doubling bifurcation occurs. Namely, it is considered that the conservation of clustering under the period doubling bifurcation holds. Experimental observation that there are 4 kinds of 2-clusterd periodic motions may also imply that at least 4 kinds of periodic attractors coexist separately in space in these parameter regions. If the bifurcation parameter is increased beyond the periodic region, the complete synchronization

among the states of all neurons seems to break in chaotic regions.
However, even if the complete (anti)synchronization does not hold in chaotic regions, it may be said that there still remains an effect of clustering in the following sense: In the chaotic regions near to the critical value of the parameter, the neurons belonging to each previous cluster remain in the same cluster and behave showing strong correlation with each other (Fig. 5 and Fig. 6).
In Fig. 5, note that correlations R_i (i=1,2,3,4) between the stored pattern and the internal states of the network are kept almost completely even in the region of chaos till the bifurcation parameter exceeds a certain critical value(see also, Table 1). We coined the term "quasi-synchronization" for these chaotic behaviors.

Table 1. The numerical data of correlations R_i (i=1,2,3,4) vs. a: R_j for quasi-synchronized state takes slightly small value from that of completely synchronized state.

a	R_1	R_2	R_3	R_4
0.683	1.000000	1.000000	1.000000	1.000000
0.685	1.000000	1.000000	1.000000	1.000000
0.687	0.998647	0.998670	0.998667	0.998670
0.689	0.991699	0.991454	0.991423	0.991529

3.3. Dynamic recombination (self-switching) of clustered patterns

If we increase the bifurcation parameter further, a chaotic attractor which exhibits a quasi-synchronized motion being specified by one of the stored patterns grows in size gradually and experiences a sudden change to a periodic attractor, and again new chaotic attractor is generated as is seen in the bifurcation diagram of Fig. 3.
By our experiment, after several repetitions of transitions between chaotic and periodic attractors, it appears a drastic change in the retrieval dynamics of the network. Beyond a certain critical value of the bifurcation parameter, it appears a new mode of chaotic dynamics, i.e., instead of the 2-clustered quasi-synchronized state previously observed, it occurs a spontaneous switching phenomena between 4 kinds of 2-clustered (quasi-synchronized) states, each of which corresponds to one of the stored patterns respectively.
Our experimental results disclose that when the process of switching from one stored pattern to another pattern will occur, it is crucial to take place a dynamic recombination of the certain constituent units among the 2-clustered states, which enables the change of network from one stored pattern to another stored memory. Typical examples of the switching phenomena are presented in Fig. 7 and Fig. 8.
The results shown in these figures are obtained from single temporal evolution of the network by observing different time durations respectively.

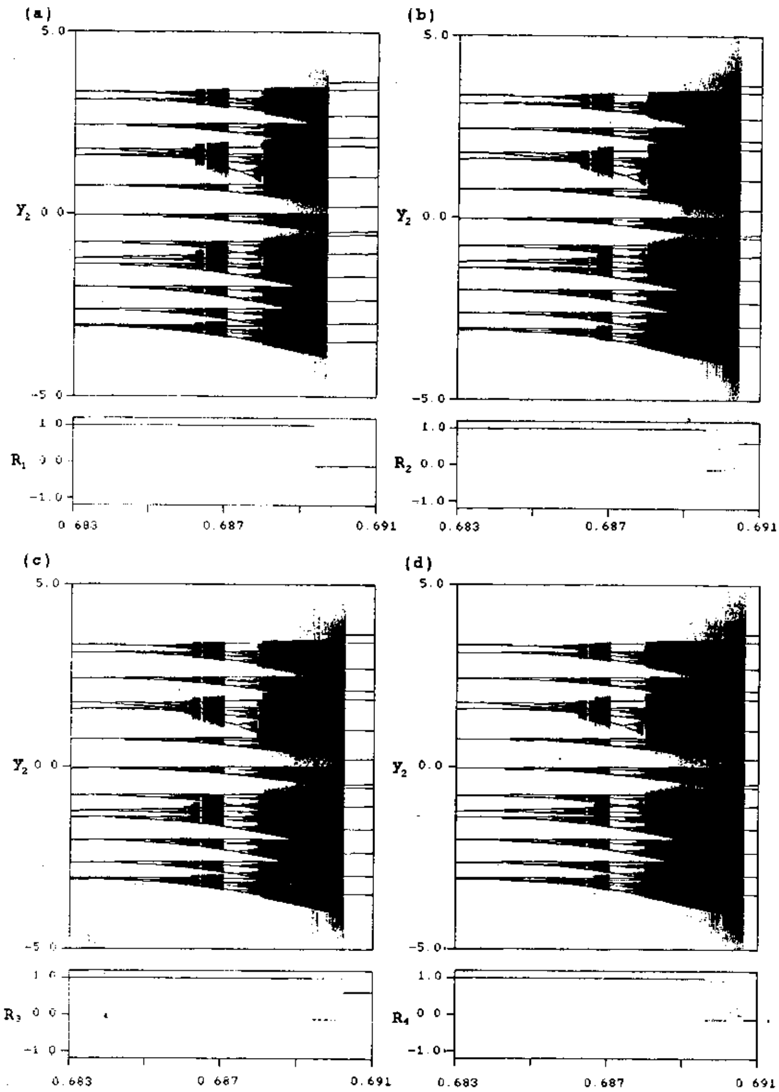


Fig. 5. The Correlation between each stored pattern and the state of the network vs. a : The Correlations R_i (i=1,2,3,4) are evaluated as follows. By using a time averaged correlations $r_{i,j}$ between the internal state of i-th and j-th neurons, we define R_i by :

$$R_1=(r_{1,3}+r_{1,5}+r_{1,7}+r_{1,9}+r_{1,11}+r_{1,13}+r_{1,15}+r_{2,4}+r_{2,6}+r_{2,8}+r_{2,10}+r_{2,12}+r_{2,14}+r_{2,16})/14$$
$$R_2=(r_{1,2}+r_{1,5}+r_{1,6}+r_{1,9}+r_{1,10}+r_{1,13}+r_{1,14}+r_{3,4}+r_{3,7}+r_{3,8}+r_{3,11}+r_{3,12}+r_{3,15}+r_{3,16})/14$$
$$R_3=(r_{1,2}+r_{1,3}+r_{1,4}+r_{1,9}+r_{1,10}+r_{1,11}+r_{1,12}+r_{5,6}+r_{5,7}+r_{5,8}+r_{5,13}+r_{5,14}+r_{5,15}+r_{5,16})/14$$
$$R_4=(r_{1,2}+r_{1,3}+r_{1,4}+r_{1,5}+r_{1,6}+r_{1,7}+r_{1,8}+r_{9,10}+r_{9,11}+r_{9,12}+r_{9,13}+r_{9,14}+r_{9,15}+r_{9,16})/14$$

In the figures, the upper figure gives the bifurcation diagram of y_2, and the lower figure shows the correlation R_i during 50,000 steps vs. a. The figures (a)-(d) are obtained by following 4-kinds of 2-clusterd motions which correspond to one of stored patterns respectively. Note that synchronized or quasi-synchronized state is preserved even if bifurcations have occurred.

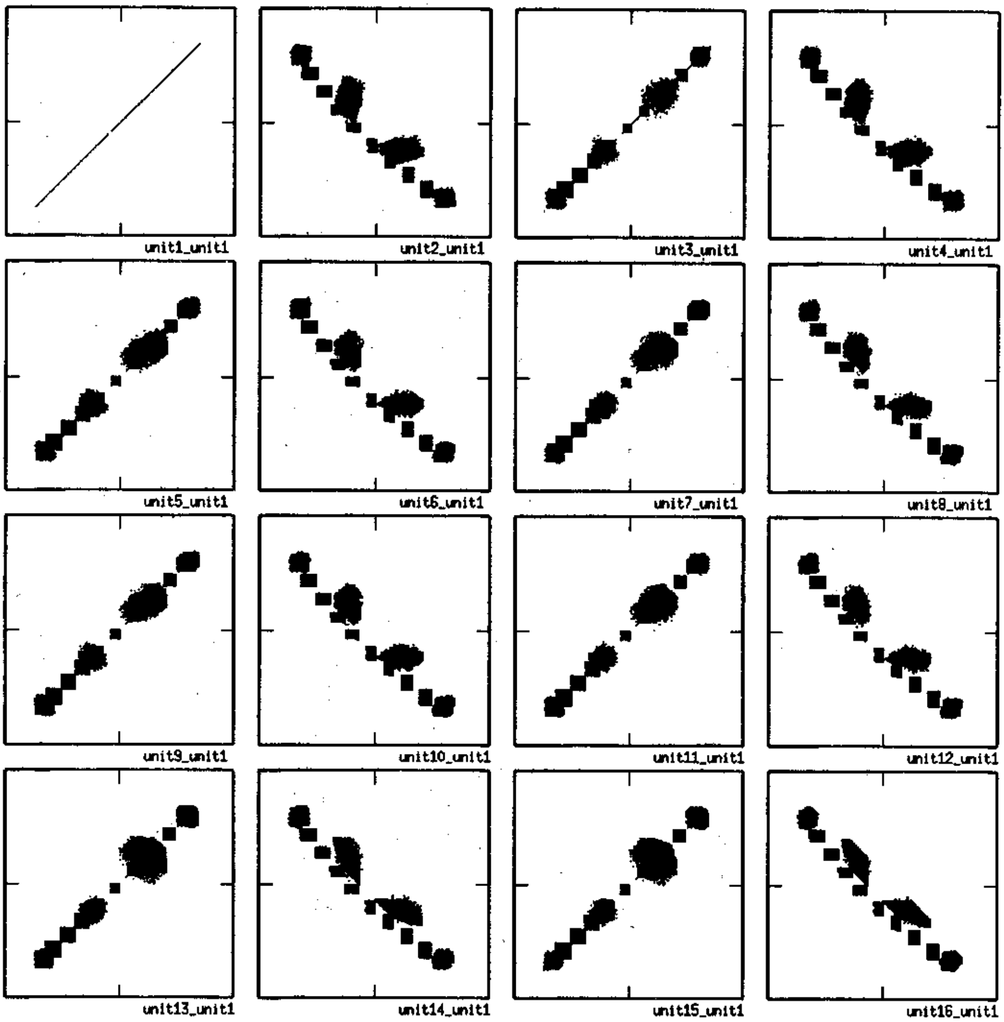


Fig. 6. The 2-dimensional plot of an chaotic orbit exhibiting a quasi-synchronized state(a=0.6888). In each figure, while horizontal axis represents the internal state of 1st neuron y_1 ($-5< y_1<5$), vertical axis represent the internal state of i-th neuron y_i ($i=1,2,\ldots,16$; $-5< y_i<5$). These figures denote a quasi-synchronization phenomena between 1st neuron and 3rd, 5th, ... , 15th neurons and anti-phased synchronization between 1st neuron and 2nd, 4th, ... , 16th neurons. The network continues to recall one of stored pattern $\mathbf{p}^1$.

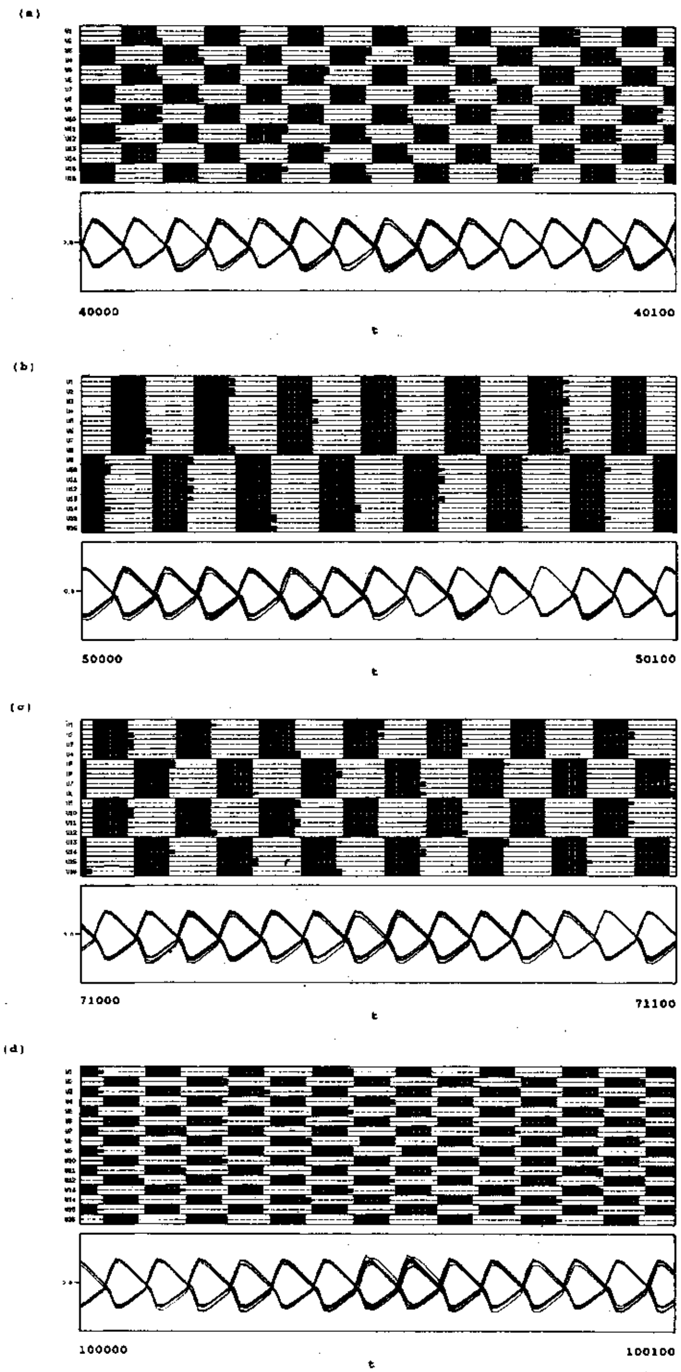


Fig. 7. Dynamic retrievals of all stored patterns (a=0.6902). In the course of temporal development, every stored patterns is retrieved in the network. Note that every stored pattern appears by following single temporal development, which shows strong contrast to retrieval characteristic in other parameter regions.

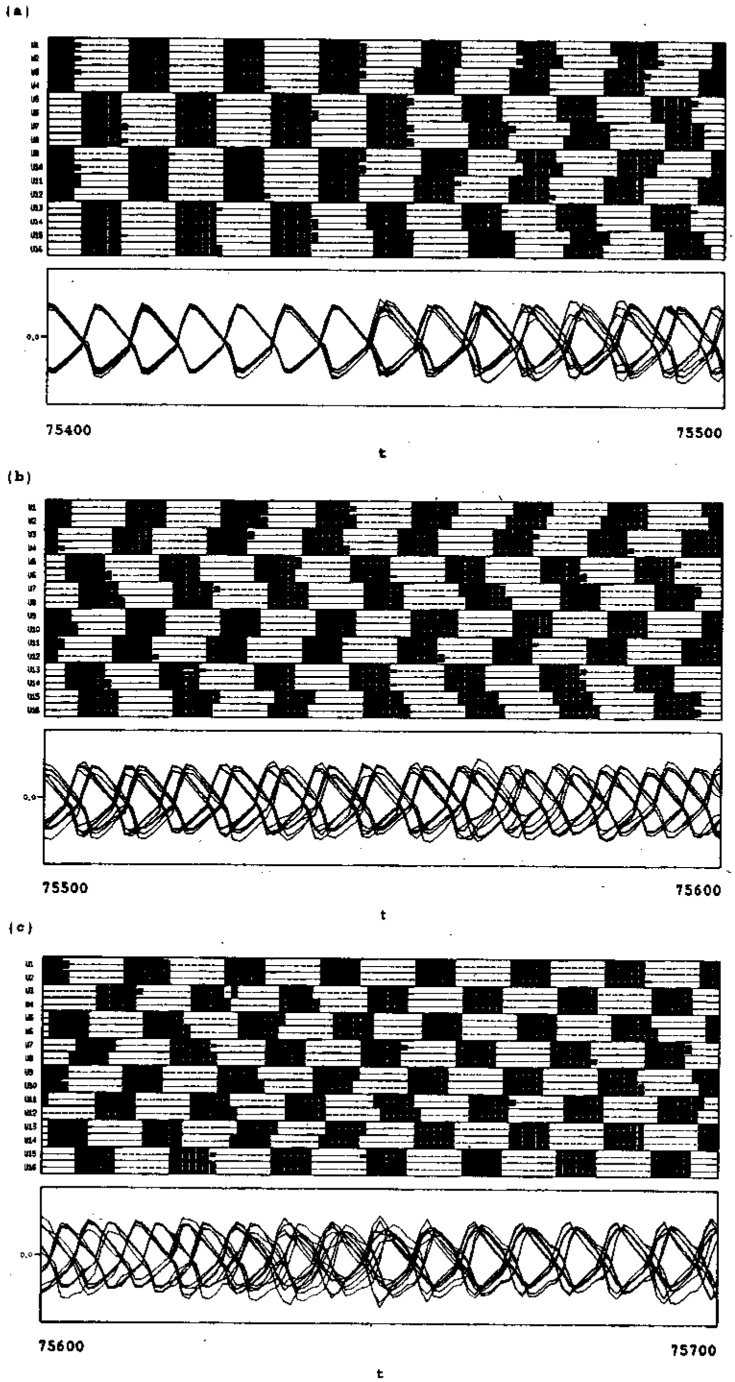
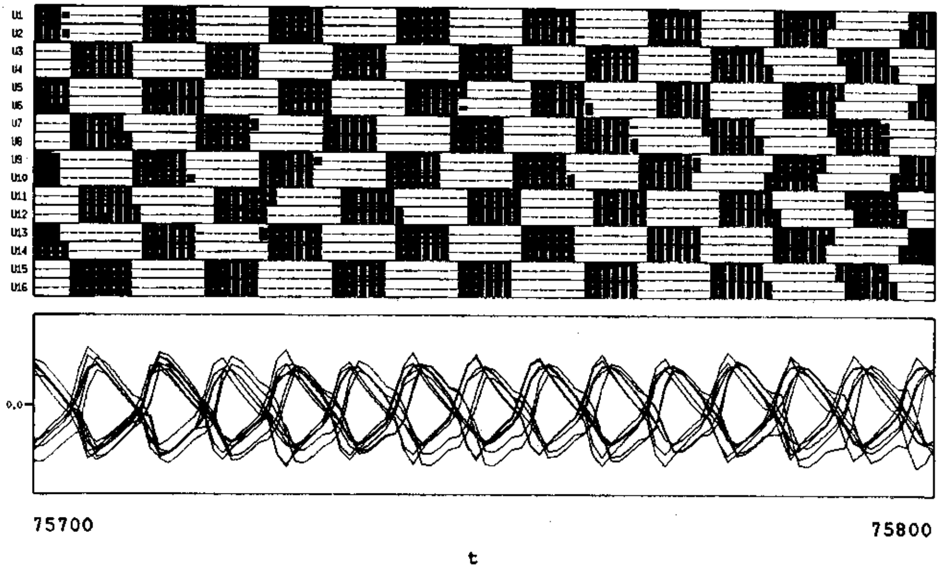
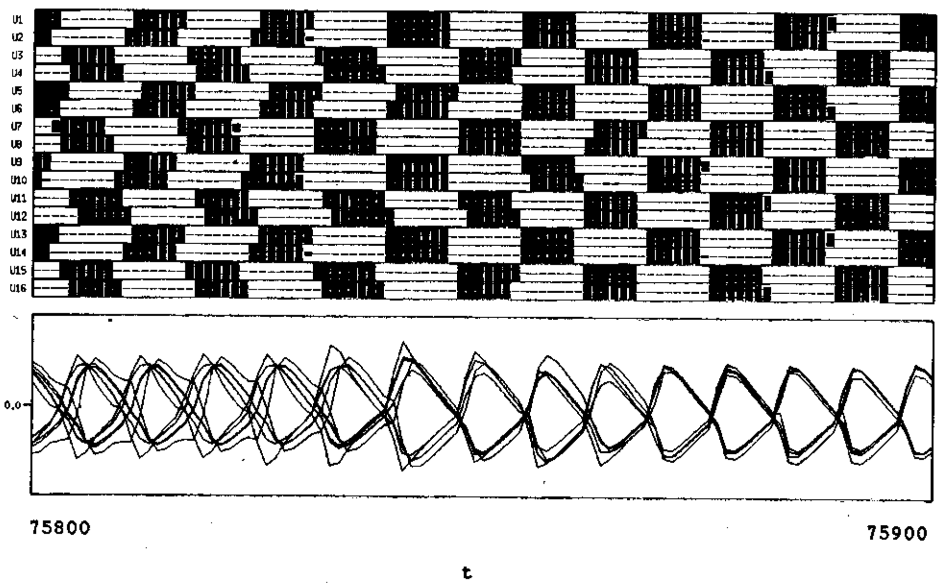


Fig. 8. The dynamic recombination of stored patterns during switching (a=0.6902). In this figure, the network first recalls one of stored patterns $\mathbf{p}^3$, then the quasi-synchronized state corresponding to the pattern $\mathbf{p}^3$ begins to break and recombination dynamics takes place toward other quasi-synchronized state corresponding to the pattern $\mathbf{p}^2$.

(d)



(e)



While the results in Fig.7 indicate that the network can retrieve every stored pattern, Fig. 8 represents an example of switching processes between different kinds of 2-

clustered quasi-synchronized states, where the transition from memory pattern $\mathbf{p}^3$ to $\mathbf{p}^2$ are taking place by recombining the certain constituent units in the corresponding clusters. In case of the switching from $\mathbf{p}^3$ to $\mathbf{p}^2$ in Fig. 8, the dynamic recombination of memory is accomplished as follows: after exiling 3, 4, 11, 12-th (5, 6, 13, 14-th) neurons from one of the two clusters representing $\mathbf{p}^3$, then they are recombined to the other clusters which represent $\mathbf{p}^2$.

3.4. Statistical property of Self-Switching and an inference for the onset mechanism

The self-switching phenomena between 2-clustered quasi-synchronized states continue sufficiently long time duration, and it turns out that all kinds of 2-clustered states corresponding to every stored pattern appear during that time duration (Fig. 9).

Table 2. A statistical property of switching. The table gives the total residence time for each stored pattern $\mathbf{p}^k$ (k=1,2,3,4) and the number of switching during 1×10^{9} steps. While every stored pattern is retrieved via switching, the frequency of switching becomes small when a is decreased from above.

a	$\mathbf{p}^1$	$\mathbf{p}^2$	$\mathbf{p}^3$	$\mathbf{p}^4$	total	number of switching
0.689000	185576503	464979818	46892373	289937737	987386431	24
0.689040	251573134	81904050	298753263	329189179	961419626	43

In Table 2, we present the result on the statistical property of switching behaviors. By our experiment, it has a tendency that the frequency of the switching becomes small by decreasing the bifurcation parameter, but it is not clear whether simple scaling law does hold or not when the bifurcation parameter is decreased from above toward the critical value. Regarding the statistical property of the switching phenomena, it should be noted that switching phenomena from one clustered state to another clustered state occurs irregularly and intermittently. Typical examples of the intermittent property of the switching is shown in Fig. 10.

From the experimental results described thus far, it is not unnatural to infer that the onset mechanism for the self-switching phenomena when the bifurcation parameter is increased from below is due to the *attractor merging crisis* where 4 kinds of previous chaotic attractors corresponding to the 4 stored patterns, each of which has occupied a localized space separately merge at the critical parameter [11].

The experimentally determined critical bifurcation values at which the switchings have occurred in each bifurcation route generated by 4 isolated attractors respectively take approximately an identical value, as shown in Table 3.

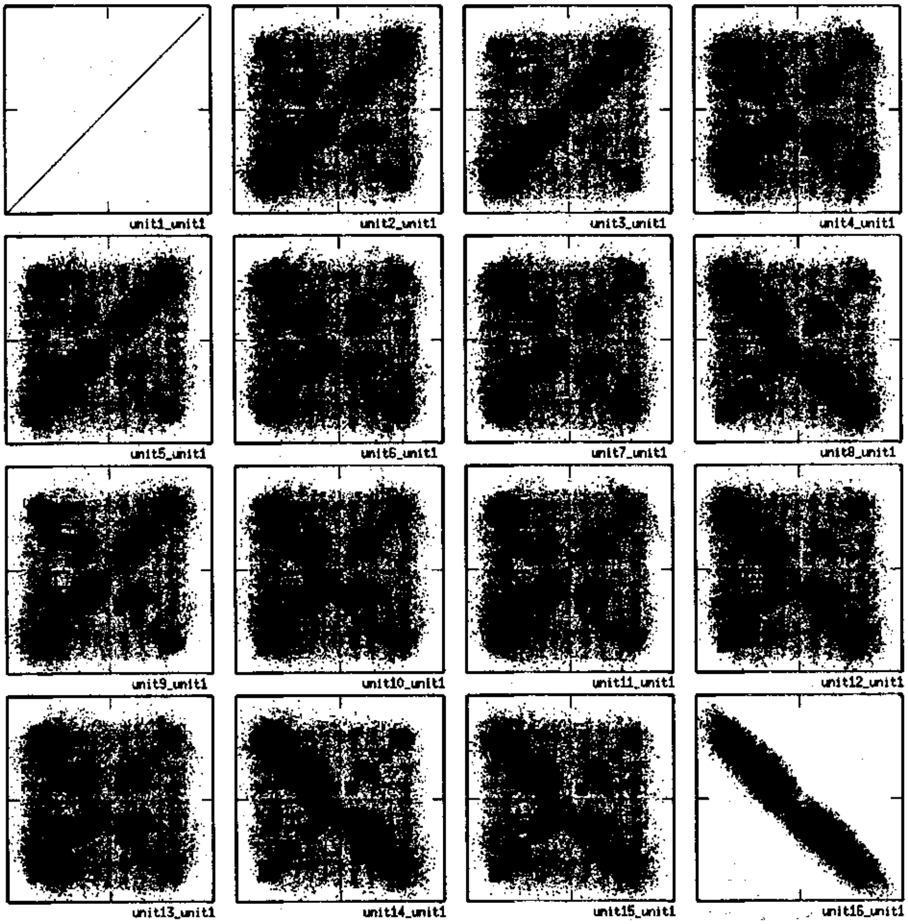


Fig. 9. Long time 2-dimensional plot of an orbit exhibiting switching (a=0.6902). The figure shows that all stored patterns have been retrieved during 300,000steps.

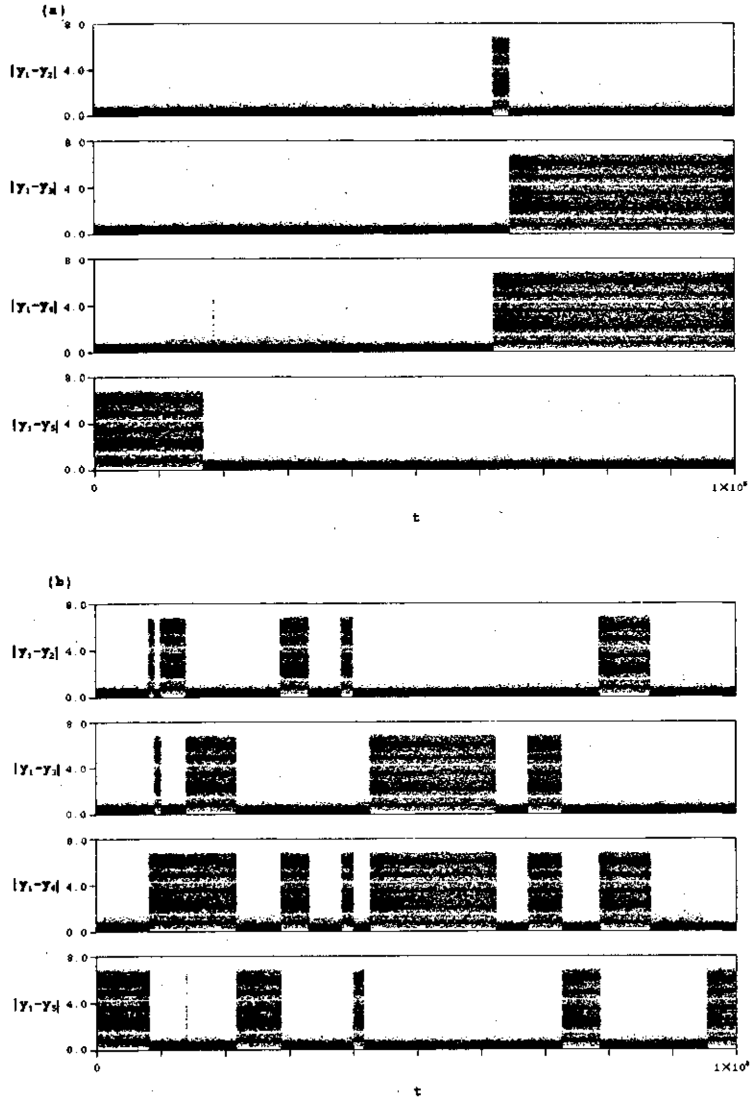


Fig. 10. An intermittent property of switching phenomena. The figures shown in (a) and (b) represent temporal developments of $|y_1\text{-}y_2|$, $|y_1\text{-}y_3|$, $|y_1\text{-}y_4|$, $|y_1\text{-}y_5|$ during 1×10^8 steps respectively. The frequency of recombinations of quasi-synchronized state at a=0.6892 (a) is less than that of at a=0.6894 (b).

Table 3. The critical values for the onset of switching. The table gives the parameters, at which the quasi-synchronized states begin to break by increasing parameter a. The critical values are estimated based on the correlation R_i (i=1,2,3,4) (see also Fig. 5).

	R_1	R_2	R_3	R_4
a	0.689380	0.689600	0.689420	0.689620

4. Summaries and discussions

Using the chaotic neural network designed for auto-associative memory, we have shown that it is possible to generate a new mode of associative dynamics of memory, i.e., every stored pattern can be retrieved in a single temporal development based on dynamic recombinations of quasi-synchronized states.

It may be possible to summarize our results as follows. Utilizing the chaotic neural network designed for associative memory, we have found a new phenomena coined by the term : dynamic recombination (Self-switching) of memories, through which one of the stored patterns are deformed dynamically to another stored pattern in the course of temporal development of the neural network. We have indicated that a basis of dynamic recombination of memories would be the generalized synchronization property of oscillations of neuronal state in the network, and an origin of the synchronization property would be the symmetry property of the network. The symmetry of our neural network for auto-associative memory is due to the coupling matrix which is determined by the set of stored memory patterns.

We hope the results presented in this paper will contribute to harnessing of chaos in developing new technology in information processing. From the point of view of engineering applications, we add the following comment.

The property of switching between every stored memory pattern discussed in this paper will be applicable to the problems of retrieving memory if some suitable adaptive or annealing algorithms would be supplemented to our dynamical system. It should be made some comments on the relation of our result to the others.

Chaotic switching phenomena have been found in various research fields [1], [16], [23], [24], [32]. While there appear a growing number of articles which have an aim to analyze and utilize chaos in electronic circuits [5], [9], [18], [27], Nishio et al have found switching phenomena in chaotic circuits coupled by resistors [24]. While Adachi and Aihara [1] initiated dynamic associative memories using a chaotic neural network, Kaneko [16], [17] has proposed the globally coupled map (GCM) and has found chaotic itenerancy phenomena, which look very similar to our experimental results. It may be noted that the globally coupled map corresponds to the system with a uniform coupling which belongs to a class of system with full symmetry, while our system has a non-trivial and specific symmetry which can be specified by stored pattern.

It remains a future important subject to investigate systematically the role of symmetry of neural network designed for associative memory in forming the clustered motions and their recombination dynamics, which are studied in a heuristic manner in this paper. Investigations on the scaling law associated with a global bifurcation of *attractor merging crisis* which is also inferred in this paper may constitute another interesting topic.

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Appendix

The set of matrix Q_i representing the permutation γ_i (i=1,2,3,4) which preserves a symmetry of dynamic system described in eqs.(1)~(3) with the coupling (5):

$$Q_1 = \begin{pmatrix}
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0
\end{pmatrix}$$

$$Q_2 = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{pmatrix}$$

$$Q_3 = \begin{pmatrix}
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0
\end{pmatrix}$$