Generalized Geometry & Topological Field Theory

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1. Introduction

目的1

Topological M theory

'04 Dijkgraaf. Gukov
Neitzke Vafa
'04 Nekrasov

What is the topological M theory?

Conjecture

Topological M theory is described by the Hitchin functionals

Superstring with nontrivial NS flux H

○ N=2 superstring with NS flux $H \neq 0$ は generalized geometry の構造を持つ

<u>worldsheet</u>

1. 2D N=(2,2) SUSY σ-model with H flux
   $H = db \neq 0$
   '84 Gates, Hull, Roček
   '04 Gualtieri

↳ 2. 1-st order formalism & SUSY TFT
   '04 Lindström, Minasian, Tomasiello, Zabzine

<u>target space</u>

3. N=2 superstring (SUGRA) with H flux
   '04 Graña, Minasian, Petrini, Tomasiello
   '03 Fidanza, Minasian, Tomasiello

○ flux compactification

2. Hitchin Functional

<u>Def</u> ベクトル空間 V

V^* 上の p-form $\rho \in \Lambda^p V^*$ が stable

$\overset{\text{def}}{\Longleftrightarrow}$ 自然な作用 $GL(V) \curvearrowright \Lambda^p V^*$ に対して

$GL(V)$ の作用が $\Lambda^p V^*$ 上 open orbit

(nondegenerate の一般化)

<u>例</u> presymplectic form ω

$\dim V = n = 2m =$ even

$\omega \in \Lambda^2 V^*$ とする

ω が stable $\Longleftrightarrow$ $\det \omega_{ij} \neq 0$

(nondegenerate)

$\because$) $\forall \omega, \omega'$ s.t. $\det \omega_{ij} \neq 0$, $\det \omega'_{ij} \neq 0$

に対して $\exists A \in GL(V)$

$\omega' = A \cdot \omega \, {}^t A$

注) n が odd のときは ω stable だが degenerate

注) p form ρ stable $\Leftrightarrow$ n-p form $*\rho$ stable

$*$ (Hodge star)

一般に $n \gg 0$ のとき

$$\dim GL(V) = n^2 \ll \dim \Lambda^p V^* = \frac{n!}{p!(n-p)!}$$

だから n が大きいとき stable form はあまりない。

Sato, Kimura

stable forms の分類

		stabilizer	$*$-dual
1. n all $p=1$	nonzero 1-form		$n-1$-form
2. n all $p=2$			
$n=2m$	nonzero 2-form	$Sp(2m, \mathbb{R})$	$n-2$-form
3. n all $p=n$			$\times$
4. $n=6, p=3$		$SU(3)\times SU(3)$	$p=3$
5. $n=7, p=3$		G_2	$p=4$
6. $n=8, p=3$	spin $\frac{3}{2}$	$PSU(3)$	$p=5$
	(spin $\frac{1}{2}$	$Spin(7)$)	

次に

例2. $\omega \in \Lambda^2 V^*$ symplectic form とする。 $n=2m=$ even のとき

自然な volume がある。(stable $\Leftrightarrow \omega^{\frac{n}{2}} \neq 0$)

$$S = \frac{1}{m!}\int_M \omega^{\frac{n}{2}}$$

☆ conformal scaling λ に対する性質

$\omega \to \lambda^2 \omega$ より $S \to \lambda^n S$

☆ $*\omega = \hat{\omega}$ とおくと $\hat{\omega} = \omega^{m-1}$

$$S = \frac{1}{m!}\int \omega \wedge \hat{\omega}$$

今 $[\hat{\omega}] \in H^{n-1}(M)$ を fix して $\hat{\omega}$ の

変分をおこなう

$[\hat{\omega}] \in H^{n-1}(M)$ より $d\hat{\omega} = 0$

よって $\hat{\omega} = \omega_0 + d\gamma$ とおくと

変分は $\delta S = \int \omega \wedge d(\delta\gamma)$

$$\sim \int d\omega \wedge \delta\gamma$$

$\Longleftrightarrow d\omega = 0$ symplectic

つまり
$d\omega=0$ から $d\hat{\omega}=0$ が
critical point として出る。
→ Sの記述する幾何 = symplectic幾何

4,5,6 に対する同様の "action" を
<u>Hitchin functional</u> という。

4. ρ : 3-form　　$n=6$, $p=3$

$$K_n{}^p \equiv \rho_{ijk}\rho_{lmn}\epsilon^{ijklmp}$$

$$S=\int\sqrt{-K_n{}^p K_p{}^n}=\int\left|\sqrt{-\mathrm{tr}K^2}\right|$$

critical point　ρ : holomorphic 3-form

$$\left[\begin{array}{lll} n=6 & p=2 & \omega \\ & p=3 & \rho \end{array}\right] \Rightarrow \text{Kähler}$$

---

② $d\omega=0$ が critical point
$\Leftrightarrow d\hat{\omega}=0$

よって $d\omega=0$
かつ $d\hat{\omega}=0$ $\Leftrightarrow$ symplectic幾何

5. $n=7$, $p=3$

ρ: 3-form

$$K_{ij} = \rho_{ik_1k_2}\,\rho_{jk_3k_4}\,\rho_{k_5k_6k_7}\,\epsilon^{k_1k_2k_3k_4k_5k_6k_7}$$

$$S = \int (\det K_{ij})^{\frac{1}{9}}$$

G_2-manifold

6. $n=8$, $p=3$

ρ: 3-form

$$d_{ij}{}^{l} = \rho_{ik_1k_2}\,\rho_{jk_3k_4}\,\rho_{k_5k_6k_7}\,\epsilon^{lk_1\sim k_7}$$

$$d_{ij}{}^{l}\,d_{pl}{}^{j} \equiv G_{ip}$$

$$S = \int (\det G)^{\frac{1}{18}}$$

$PSU(3)$

これを一般化できる。

ρ : p-form $\to$ ρ : multi-form

$$\begin{cases} p : \text{even のとき} & \rho = \rho_0 + \rho_2 + \rho_4 + \cdots \\ p : \text{odd のとき} & \rho = \rho_1 + \rho_3 + \rho_5 + \cdots \end{cases}$$

として Hitchin functional を作く。

積分は n-form part を積分

これを extended Hitchin functional

$\to$ This define generalized geometry

Hitchin functional = topological M ?

$\boxed{\text{evidence}}$

1. topological field theory

2. 次元　stabilizer = holonomy

6	$SU(3)\times SU(3)$	superstring
7	G_2	M theory
8	$(Spin(7))$	F theory

3. tree level で

$S_{\substack{n=6\\p=2}} \leftrightarrow$ A-model

$S_{\substack{n=6\\p=3}} \leftrightarrow$ B-model

4. $S_{n=7}$

$\downarrow$ dimensional reduction

topological gravity 6D, 4D, 3D, 2D

'05 Pestun, Witten ~~10~~

But!

"1-loop"

- Hitchin fnal $\neq$ B-model
- extended Hitchin fnal = B-model

generalized geometry is needed.

3. Generalized Geometry
(generalized complex structure)

2種類のdefのしかた。

① p-form ρ 上の作用として定義
→ pure spinor target sp rep. Hitchin

② 作用の"交換関係"で定義 Gualtieri
→ world sheet rep. Courant bracket

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②で定義する。

cf. complex structure J

n-dim manifold M 上

$J : TM \to TM$

s.t. $J^2 = -1$ ——①

$\pi_{\mp}[\pi_{\pm}X, \pi_{\pm}Y] = 0$ ——②

ここで X, Y : vector field on TM

$\pi_{\pm}$: projection on $\pm\sqrt{-1}$ eigen bundle

$[\ ,\ ]$: Lie bracket

M in-dim manifold

$TM \oplus T^*M$ を考える

X, Y; vector field on TM

ξ, η: 1-form on T^*M

$X+\xi \in \Gamma(TM \oplus T^*M)$

section on $TM \oplus T^*M$

◎ Inner product $\langle \, , \, \rangle$

$$\langle X+\xi, Y+\eta \rangle = \frac{1}{2}(i_X \eta + i_Y \xi)$$

i_X, i_Y: は interior product

つまり $d\phi^i\left(\frac{\partial}{\partial \phi^j}\right) = \delta^i{}_j$

∘ signature (n, n)

∘ 直交座標では $I = \begin{pmatrix} 0 & 1_d \\ 1_d & 0 \end{pmatrix}$

◎ Courant bracket

$$[X+\xi, Y+\eta] \equiv [X, Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2}d_M(i_X\eta - i_Y\xi)$$

$\mathcal{L}_X, \mathcal{L}_Y$: Lie微分
d_M : M上の外微分

○ 反対称
○ Jacobi identity をみたさない

注) ⟺ Dorfman bracket

$$(X+\xi)\circ(Y+\eta) \equiv [X, Y] + \mathcal{L}_X\eta - i_Y d\xi$$

○ 対称でない
○ Jacobi identity をみたす。

Generalized Complex Structure $\mathcal{J}$

$$\mathcal{J} : TM \oplus T^*M \to TM \oplus T^*M$$

s.t. $\langle \, , \, \rangle$ を保つ $\Leftrightarrow \mathcal{J}^* I \mathcal{J} = I$ —— (0)

$$\mathcal{J}^2 = -1 \qquad \text{—— (1)}$$

$$\pi_{\mp}[\pi_{\pm}(X+\xi), \pi_{\pm}(Y+\eta)] = 0 \qquad \text{—— (2)}$$

where

- $\pi_{\pm} \equiv \frac{1}{2}(1 \mp \sqrt{-1}\mathcal{J})$

  $\pm\sqrt{-1}$ eigenbundle への projection

- $[\, , \,]$ Courant bracket

$$\mathcal{J} = \begin{array}{cc} \begin{pmatrix} J & P \\ Q & -J^* \end{pmatrix} & \begin{array}{c} TM \\ T^*M \end{array} \\ \begin{array}{cc} TM & T^*M \end{array} & \end{array}$$

とおく。

⊚ local coordinate expression

$$\mathcal{J} = \begin{array}{ccc} \begin{pmatrix} J & P \\ Q & -J^* \end{pmatrix} & \begin{matrix} TM \\ T^*M \end{matrix} \\ \begin{matrix} TM & T^*M \end{matrix} & \end{array}$$

ⓒ ① $J^i{}_k J^k{}_j + P^{ik} Q_{kj} + \delta^i{}_j = 0$

$$J^i{}_k P^{kj} + J^j{}_k P^{ki} = 0$$

$$Q_{ik} J^k{}_j + Q_{jk} J^k{}_i = 0$$

$$P^{ij} + P^{ji} = 0, \quad Q_{ij} + Q_{ji} = 0$$

② $A^{ijk} = B_i{}^{jk} = C_{ij}{}^k = D_{ijk} = 0$

$$A^{ijk} = P^{il}\partial_l P^{jk} + (ijk \text{ cyclic})$$

$$B_i{}^{jk} = J^l{}_i \partial_l P^{jk} + P^{jl}(\partial_i J^k{}_l - \partial_l J^k{}_i) + P^{kl}\partial_l J^j{}_i - \partial_i J^j{}_l P^{lk}$$

$$C_{ij}{}^k = J^l{}_i \partial_l J^k{}_j - J^l{}_j \partial_l J^k{}_i - J^k{}_l \partial_i J^l{}_j + J^k{}_l \partial_j J^l{}_i + P^{kl}(\partial_l Q_{ij} + \partial_i Q_{jl} + \partial_j Q_{li})$$

$$D_{ijk} = J^l{}_i(\partial_l Q_{jk} + \partial_k Q_{lj}) + J^l{}_i(\partial_l Q_{ki} + \partial_i Q_{lk}) + J^l{}_k(\partial_l Q_{ij} + \partial_j Q_{li}) - Q_{jl}\partial_i J^l{}_k - Q_{kl}\partial_j J^l{}_i - Q_{il}\partial_k J^l{}_j$$

② Automorphism on GCS
semi-direct product of
Diff(M) & b-transformation
(b-field transf.)

Def b-transformation ($\mathrm{Diff}(M) \ltimes \Omega^2_{closed}(M)$)
for 2-form $b = b_{ij} d\phi^i d\phi^j$
$$\exp(b)(X+\xi) \equiv X + \xi + i_X b$$

1. If $d_M b = 0 \Rightarrow$
$$[\exp(b)(X+\xi), \exp(b)(Y+\eta)] = \exp(b)[X+\xi, Y+\eta] \quad \text{covariant}$$

2. $\hat{\mathcal{J}} \equiv \exp(-b)\,\mathcal{J}\exp(b)$ adjoint

∘ local coordinate expression
$$\begin{cases} \hat{J}^i{}_j = J^i{}_j - P^{ik} b_{kj} \\ \hat{P}^{ij} = P^{ij} \\ \hat{Q}_{ij} = Q_{ij} + b_{ik} J^k{}_j - b_{jk} J^k{}_i + P^{kl} b_{ki} b_{lj} \end{cases}$$

• Example

1. $\mathcal{J} = \begin{pmatrix} J & 0 \\ 0 & -J^* \end{pmatrix}$

$\mathcal{J}$ is GCS $\Leftrightarrow$ J is complex str.

2. $\mathcal{J} = \begin{pmatrix} 0 & -Q^{-1} \\ Q & 0 \end{pmatrix}$

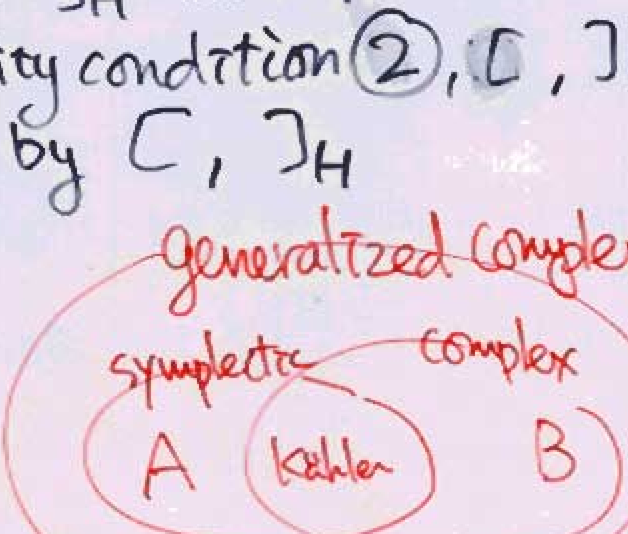
$\mathcal{J}$ is GCS $\Leftrightarrow$ Q is symplectic str.

◎ twisted Generalized Complex structure

We can generalize GCS by a closed 3-form H

$$[X+\xi, Y+\eta]_H \equiv [X+\xi, Y+\eta] + i_X i_Y H$$

In integrability condition ②, $[\ ,\ ]$ is replaced by $[\ ,\ ]_H$



○ $d_M b \neq 0$ となる b に対して

$$[\exp(b)(X+\xi), \exp(b)(Y+\eta)]_{H-d_M b}$$

$$= \exp(b)[X+\xi, Y+\eta]_H$$

つまり b-transformation で $\hat{H} = H - d_M b$

とずれる。tGCS は $H \in H^3(M)$ で決まる。

② $A_H{}^{ijk} = A^{ijk}$

$$B_{Hi}{}^{jk} = B_i{}^{jk} + P^{il}P^{km}H_{ilm}$$

$$C_{Hij}{}^k = C_{ij}{}^k - J^l{}_i P^{km} H_{jlm} + J^l{}_j P^{km} H_{ilm}$$

$$D_{Hijk} = D_{ijk} - H_{ijk} + J^l{}_i J^m{}_j H_{klm} + J^l{}_j J^m{}_k H_{ilm} + J^l{}_k J^m{}_i H_{jlm}$$

◎ generalized Kähler structure

$\mathcal{J}_1, \mathcal{J}_2$を2つのGCSとする。

$G \equiv -\mathcal{J}_1\mathcal{J}_2$が$M$のpositive definite metricのときMをgeneralized Kähler mfdという。

例　(g, J, ω) Kählerのとき

$$\mathcal{J}_1 = \begin{pmatrix} J & 0 \\ 0 & -J^* \end{pmatrix}, \mathcal{J}_2 = \begin{pmatrix} 0 & \omega^{-1} \\ \omega & 0 \end{pmatrix}$$

とすると、$G = \begin{pmatrix} 0 & g^{-1} \\ g & 0 \end{pmatrix}$となりgeneralized Kähler

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◎ generalized Calabi-Yau

generalized Kähler

(+ (holomorphically) trivial canonical bundleをもつ)

もしくは

1-st Chern classに対する条件

4. 2D $N=(2,2)$ supersymmetric σ-model with $H=d_M b\neq 0$

○ 作り方

$N=(1,1)$ SUSY σ-model

$$S=\frac{1}{2}\int_\Sigma d^2\sigma d^2\theta\left(g_{ij}(\Phi)+b_{ij}(\Phi)\right)D_+\Phi^i D_-\Phi^j$$

$\Phi^i=\phi^i+\theta^+\psi_+^i+\theta^-\psi_-^i+\theta^-\theta^+F^i$

$N=(1,1)$ superfield

$\phi^i:\Sigma\to M$

$H=d_M b\neq 0$

$D_\pm=\frac{\partial}{\partial\theta^\pm}+i\theta^\pm\partial_\pm$ superderivative

○ manifest SUSY

$\delta_1\Phi^i\equiv\epsilon_1^+D_+\Phi^i+\epsilon_1^-D_-\Phi^i$

もう1つのSUSY

$\delta_2\Phi^i\equiv\epsilon_2^+D_+\Phi^j J_{+j}^i(\Phi)+\epsilon_2^-D_-\Phi^j J_{-j}^i(\Phi)$

理論が δ_2-SUSY を持つように $(g,b,J_\pm)$ の条件式をかく。

・$H=0$ のとき

M iKähler $\Longrightarrow N=(2,2)$ SUSY

$J_\pm$: complex str. $\omega_\pm = gJ_\pm$: Kähler form

・$H\neq 0$ のとき

'84 Gates, Hull, Roček

① $J_\pm$ は TM 上 integrable almost complex str.

② g は J_+, J_- の両方に対して hermitian

③ $\nabla^{(\pm)}_i J_\pm{}^j{}_k \equiv \partial_i J_\pm{}^j{}_k - \Gamma_\pm{}^j_{il} J_\pm{}^l{}_k - \Gamma_\pm{}^l_{ik} J_\pm{}^j{}_l = 0$

$$\Gamma_\pm{}^i_{jk} \equiv \Gamma^i_{jk} \pm \frac{1}{2} g^{il} H_{ljk}$$

$$\Gamma^i_{jk} = \frac{1}{2} g^{il} (\partial_j g_{lk} + \partial_k g_{jl} - \partial_l g_{jk})$$

一般に $\omega_\pm = gJ_\pm$ は closed で無い。

これを bi-Hermitian str. という

$\Longleftrightarrow$ (twisted) generalized Kähler structure

'04 Gualtieri

・対応

$$\mathcal{J}_{1/2} \equiv \frac{1}{2}\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}\begin{pmatrix} J_+ \pm J_- & -(\omega_+^{-1} \mp \omega_-^{-1}) \\ \omega_+ \mp \omega_- & -({}^tJ_+ \pm {}^tJ_-) \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -b & 1 \end{pmatrix}$$

where $\omega_\pm = gJ_\pm$

5 Topological Sigma Model with tGCS

目的
world sheet (world volume) description of the Hitchin functionals (& topological M theory)

Hint
1. 2D $N=(2,2)$ SUSY σ-model with $H = d_M b \neq 0$
$\downarrow$ topological twist
topological σ-model

'03 Kapustin
'04 Kapustin Li

- topological twist
$\underbrace{Q_1, Q_2}_{\text{SUSY}}, \underbrace{I_V, I_A}_{2\text{つの}U(1)}$ を組み合わせてTFTをつくる

I_V, I_A にanomalyが無いことが必要
$\Longleftrightarrow$ M: generalized CY のときだけ可能

J_+ の $\pm\sqrt{-1}$ eigenbundle を $T_+^{1,0} \oplus T_+^{0,1}$

J_- 〃 $T_-^{1,0} \oplus T_-^{0,1}$

とする

$TM \otimes \mathbb{C} \cong T_+^{1,0} \oplus T_+^{0,1} \cong T_-^{1,0} \oplus T_-^{0,1}$

anomaly cancellation condition

$U(1)_V : c_1(T_-^{1,0}) - c_1(T_+^{1,0}) = 0$

$U(1)_A : c_1(T_-^{1,0}) + c_1(T_+^{1,0}) = 0$

$\Leftrightarrow$ generalized Calabi-Yau condition

$c_1(\ )$: 1-st Chern class

BRST charge

$$Q_B \sim g_{ij}\partial_+\phi^i(1+\sqrt{-1}J_+)\psi^j_+ + g_{ij}\partial\phi^i(1+\sqrt{-1}J_-)\psi_-^j$$

cf. $J_+ = -J_-$ のとき A-model

$J_+ = J_-$ のとき B-model

2. topological σ-model on the symplectic mfd

$$S = \int_\Sigma \frac{1}{2}\Omega_{ij}\,d\phi^i d\phi^j$$ '89 Witten

3. topological σ-model on the Poisson mfd

$$S = \int_\Sigma B_i d\phi^i + \frac{1}{2}P^{ij}B_i B_j$$ '93 N. I. Izawa
'94 Schaller Strobl

$$\mathcal{J} = \begin{pmatrix} J & P \\ Q & -J^* \end{pmatrix}$$

$1+2+3 \longrightarrow$

$\phi^i : \Sigma \to M$

$B_i \in T^*\Sigma \otimes \phi^*(TM)$ 1-form

d : exterior derivative on Σ

Zucchini '04

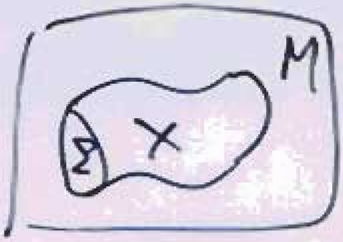
$$\boxed{S_Z = \int_\Sigma B_i d\phi^i + J^i{}_j B_i d\phi^j + \frac{1}{2} P^{ij} B_i B_j + \frac{1}{2} Q_{ij} d\phi^i d\phi^j}$$

Hitchin σ-model (Zucchini model)

J, P, Q is GCS $\underset{\not\Leftarrow}{\Rightarrow}$ S_Z is BRST inv.

∘ twisted GCS

X : 3D mfd s.t. $\partial X = \Sigma$



$$S'_Z = S_Z + \int_X \phi^* H \quad \text{WZ-term}$$

J, P, Q_{iH} is twisted GCS $\underset{\not\Leftarrow}{\Rightarrow}$ S'_Z : BRST inv.

∘ Problems

∘ GCS is <u>not</u> equivalent to BRST str.

∘ Model is <u>not</u> invariant under b-transformation.

b-transformation

$$\begin{cases} \hat{\phi}^i = \phi^i \\ \hat{B}_i = B_i + b_{ij} d\phi^j \end{cases}$$

then $$\hat{S}_2 = S_2 - \int_\Sigma b_{ij} d\phi^i d\phi^j$$

② Solution

3D world volume is more natural.

because

Courant bracket $[X+\xi, Y+\eta]$

The algebra constructed from the Courant bracket is Courant algebroid

and

Moduli of Courant algebroids

$\cong$ 3D Schwarz type TFT moduli N.I'02
(BF theory)

cf.) 2D Schwarz type TFT moduli

$\cong$ Lie algebroid moduli

Courant algebroid $\otimes \mathbb{C}$

$\downarrow$ $\mathcal{J} = \pm\sqrt{-1}$ projection

GCS

<u>therefore</u>

3D Schwarz type TFT (Courant σ-model)

$\downarrow$ $\mathcal{J} = \pm\sqrt{-1}$ projection

3D generalized complex σ-model

$\downarrow$ reduction or boundary action

2D generalized complex σ-model

3D abelian BF theory $X \to M$ [27]

X^t

$\phi^i : X \to M$ scalar 0-form

$A^i \in T^*X \otimes \phi^*(T^*M)$ 1-form

$B_{1i} \in T^*X \otimes \phi^*(TM)$ 1-form

$B_{2i} : \wedge^2 T^*X \otimes \phi^*(TM)$ 2-form

free BF action

$$S_0 = \int_X -\underset{2}{B_{2i}}\, \underset{0}{d\phi^i} + \underset{1}{B_{1i}}\, \underset{1}{dA^i}$$

interaction term 𝔈𝜆𝔥𝔷

$$S = S_0 + S_{int}$$

most general interaction

$$S_{int} = \int_X f_{1i}{}^j(\phi) B_{2j} A^i + f_2{}^{ij}(\phi) B_{2i} B_{1j}$$
$$+ \frac{1}{3!} f_{3\,ijk}(\phi) A^i A^j A^k + \frac{1}{2} f_{4\,ij}{}^k(\phi) A^i A^j B_{1k}$$
$$+ \frac{1}{2} f_{5\,i}{}^{jk}(\phi) A^i B_{1j} B_{1k} + \frac{1}{3!} f_6{}^{ijk}(\phi) B_{1i} B_{1j} B_{1k}$$

Sがゲージ不変(BRST不変)

より $f_1 \sim f_6$ に条件式が出る

$\Leftrightarrow$ $f_1 \sim f_6$ が Courant algebroid

の structure functions

次にこの model で

$$f_1 = -J \qquad f_4 = -PH + d_M J$$

$$f_2 = -P \qquad f_5 = d_M P$$

$$f_3 = JH + d_M Q \qquad f_6 = 0$$

とおくと、

$$S_{GC} = \int_X -B_{2i} d\phi^i + B_{1i} dA^i$$

$$- J^i{}_j B_{2i} A^j - P^{ij} B_{2i} B_{1j}$$

$$+ \frac{1}{2}\left(J^l{}_i H_{jkl} + \frac{\partial Q_{jk}}{\partial \phi^i}\right) A^i A^j A^k$$

$$+ \frac{1}{2}\left(-P^{kl} H_{ijl} - \frac{\partial J^k{}_j}{\partial \phi^i} + \frac{\partial J^k{}_i}{\partial \phi^j}\right) A^i A^j B_{1k}$$

$$+ \frac{1}{2} \frac{\partial P^{jk}}{\partial \phi^i} A^i B_{1j} B_{1k}$$

3D Generalized Complex Sigma Model

$$= \int_X -\langle 0 + B_2, d(\phi+0)\rangle + \frac{1}{2}\langle A+B_1, d(A+B_1)\rangle$$

$$-\langle 0+B_2, \mathcal{J}(A+B_1)\rangle$$

$$-\frac{1}{2}\langle A+B_1, \left[\begin{pmatrix} HA & 0 \\ 0 & 0 \end{pmatrix}\mathcal{J} + A^i \frac{\partial \mathcal{J}}{\partial \phi^i}\right](A+B_1)\rangle$$

実際

。S_{GC}がゲージ不変(BRST不変)

$\Leftrightarrow$ J, P, Q, H が (twisted) GCS

ただし $A_H = B_H = C_H = \cancel{\mathcal{D}_H} = 0$

$d_M \mathcal{D}_H$

なぜなら

$\mathcal{D}_H = \mathcal{D} - H + \cdots$ なので b-変換によって

$H \to H - d_M b$ だから $\mathcal{D}_H \to \mathcal{D}_H - d_M(*)$

とずれる。よって tGCSとしては $H \in H^3(M)$

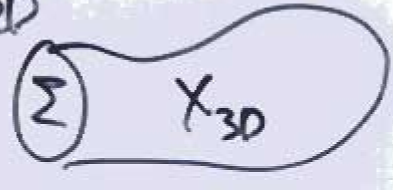
に対応して $\mathcal{D}_H \in H^3(M)$

。さらに $\hat{\phi}^i = \phi^i$, $\hat{B}_{1i} = B_{1i} + b_{ij} A^j$

$\hat{A}^i = A^i$, $\hat{B}_{2i} = B_{2i} - \frac{1}{2}\frac{\partial b_{jk}}{\partial \phi^i} A^j A^k$

と定義すると b-transformation不変 $\hat{S}_{GC} = S_{GC}$

6. 3D→2D



$\Sigma = \partial X$

Σ:2D, X:3D とする。

○ 3D Modelの boundary action として
2D Model を出す。

cf. Chern-Simons theory → WZW

H=0のとき

$$S_{GC}(X) = S_{\Sigma}(\Sigma) + S_a(X)$$

我々の3D Model　Zucchini

ここで
$$S_a(X) \equiv \int_X -Y'_{2i} d\phi^i + dB_{1i} Z'^i + Y'_{2i} A'^i + B_{2i} Z'^i + B_{2i} A'^i$$

Y'_{2i}, Z'^i 補助場

GCSによらない topological term

よって このsettingで
S_{GC} と S_{Σ} の GCS は同じ

$H \neq 0$ のとき

同様にして

$$S'_{GC}(X) = S'_Z(\Sigma) + S'_a(X)$$

ここで、

$$S'_Z(\Sigma) = S_Z(\Sigma) + \boxed{\frac{1}{2}\int_X J^l{}_i H_{jkl}\, d\phi^i d\phi^j d\phi^k - P^{kl} H_{ijl} B_{lk}\, d\phi^i d\phi^j}$$

b-invariant WZ term

Zucchini の WZ term と違う

∘ key point

S_Z & S_a is not b-invariant.

$S_{GC} = S_Z + S_a$ is b-invariant.

So $S_{GC} = S_Z(\Sigma) + S_a(X)$

$\quad \Vert$ b-transformation

$\hat{S}_{GC} = \hat{S}_Z(\Sigma) + \hat{S}_a(X)$

今 S_{Σ} (H のaction)で

b-transformation

$$\hat{S}_2(\Sigma) = S_2(\Sigma) - \frac{1}{2}\int_{\Sigma} b_{ij}\, d\phi^i d\phi^j$$

$H = -d_M b$ とおくと

$$= S_2(\Sigma) + \frac{1}{2}\int_X H$$

= Zucchini Model with H

5 Summary

3D topological σ-model
with twisted GCS on X

$\downarrow$ ∂X boundary

2D topological σ-model
with twisted GCS on ∂X

- BRST $\Leftrightarrow$ tGCS
- b-transformation invariant

topological σ-model Hitchin funals

- worldsheet description $\leftrightarrow$ spacetime description (volume)

Pestun, Witten '05

- (topological) σ-model with H-flux

6D generalized SU(3)

7D " G_2

8D " Spin(7)

'05 Witt

related to Hitchin

generalized geometry

(topological) M, F theory

- brane

σ-model with boundary