Poverty traps, the money growth rule, and the stage of financial development

(Online Working Paper)

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Abstract
This paper investigates how monetary policy influences the emergence of local indeterminacy, local bifurcations, and multiple steady states, depending upon the degree of the commitment parameter that defines financial market imperfection, using Diamond’s overlapping generations model with credit market frictions. The analytical results will show that poverty traps happen as an inevitable outcome under a wider range of money growth rates, because financial markets are less developed. Put differently, we derive analytically the positive link between financial development and per capita income.

Journal of Economic Literature Classification Number: C–62, E–32, O–42

Key Words: Poverty traps, monetary policy, financial development, indivisible investment, multiple steady states, local indeterminacy and Neimark-Sacker and saddle-node bifurcations.

1. Introduction

It is well known that there are sharply differing opinions regarding the importance of the development of financial markets to economic performance. For example, Schumpeter (1912) argued that financial markets are essential for economic development. Similarly, Hicks (1969) declared that it was a critical and inextricable part of industrialization in England. In contrast, Robinson (1952) contended that financial development responds passively to economic growth. Lucas (1988) also asserted that the role of financial factors in economic development should not be overstressed. However, a growing body of empirical work demonstrates a strong positive link between the functioning of financial markets and long-run economic growth. Empirical evidence makes it difficult to conclude that the financial system is inconsequential to the process of economic growth. See King and Levine (1993) and Levine (1993).
This paper explores how monetary policy affects the emergence of indeterminacy, local bifurcations, poverty traps and multiple steady states, depending on the degree of the commitment parameter that defines credit market frictions. If business entrepreneurs can borrow up to the present value of project revenue at the commencement of a project, there exists no credit market friction and the credit market is competitive. However, the present paper explicitly takes up credit market frictions in the sense that an entrepreneur is allowed to borrow only up to a fraction of the present value of project revenue. The lower the financial market imperfection, the larger is the proportion of the present value of project revenue up to which agents can borrow. The magnitude of credit market frictions corresponds negatively to the degree to which financial markets develop in the economy. We will show that poverty traps occur as an inevitable consequence, under a wider range of money growth rates, because financial markets are less developed. Poverty and slow development are more likely in an economy with a less developed financial system. We can say that poverty traps occur as an inevitable outcome, when either of the following two situations emerges.

1) A low-capital-stock steady state exists, but middle- and high-capital-stock steady states do not exist.
2) Low- and middle-capital-stock steady states exist, but there is no equilibrium path converging to a middle-capital-stock steady state.

To derive the positive analytical link between the functioning of financial markets and economic development, we introduce money as a substitutable asset in Diamond’s overlapping generations model with credit market frictions, as studied in Matsuyama (2004). This type of growth model has been extensively utilized in existing literature. For example, Matsuyama (2004) considered a nonmonetary version of Diamond’s growth model with credit market imperfections and analyzed the effects of financial market globalization on the inequality of nations. However, he did not explore the existence of endogenous fluctuations driven by agents’ self-fulfilling expectations and endogenous cycles through a Neimark-Sacker bifurcation, as discussed here. Using a monetary version of Diamond’s overlapping generations model, Boyd and Smith (1998) and Huybens and Smith (1998, 1999) illustrated the possibility that poverty traps are created as the inevitable outcome of a moderately high rate of inflation.

However, the present paper differs significantly from Boyd et al (1998) and Huybens et al (1998, 1999) in several respects. First, the structure of their model is analytically complex because the credit market imperfection is endogenously derived through the solution of a costly state verification problem. Due to this analytical complexity, they must rely on numerical simulations to examine the local stability of steady states and cannot emphasize the importance of financial development in investigating how likely it is that the economy is trapped at a low level of real activity. In contrast, because the structure of credit market imperfections in this paper is much simpler, we can
characterize more fully the equilibrium dynamics that demonstrate the occurrence of poverty traps as an inevitable outcome of the low stages of financial development.

There is a significant literature deriving the positive link between the functioning of financial markets and economic development (see Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Atje and Jovanovic (1993), and Greenwood and Smith (1997)), in which financial intermediation enhances growth by allowing a larger proportion of investment to be directed to activities with high returns. In the above papers, however, the importance of financial markets for macroeconomic instability is not emphasized. In contrast, the present paper examines the relations between the development stage of financial markets and the occurrence of macroeconomic instability as in poverty traps, local indeterminacy, Neimark-Sacker bifurcations, and multiple steady states.¹

The rest of the paper proceeds as follows. Section 2 describes the structure of the model, while the existence and local stability of steady states with and without credit market frictions are analyzed in section 3. In section 4, we explore how the money growth rate influences the existence of steady states and the occurrence of local indeterminacy and endogenous cycles. Using the results in section 4, section 5 explores how financial development interacts with the appearance of poverty traps as an inevitable pattern. Section 6 considers the global dynamics to enhance the representation of local dynamics analyzed in the previous sections. Section 7 concludes the paper.

2. The model

2-1. Environment

We consider an economy populated by an infinite sequence of two-period-lived overlapping generations. It can be regarded that the representative young and old agents coexist at every period, since each generation is identical and is normalized to one. Time is indexed by \( t = 0,1,\cdots \). At each date, a single final good is produced, using \( K \) units of capital and \( N \) units of labor. For analytical tractability, we focus on the case of unit-elastic capital–labor substitution given by 
\[
F(K, N) \equiv Nf(k), \quad k \equiv K / N, \quad f(k) = Ak^s \quad \text{and} \quad s \in (0,1).
\]
Capital and labor are traded in competitive markets at each date. Letting \( w_t \) and \( \rho_t \) denote the factor rewards for labor and capital, respectively, we can obtain the standard factor pricing relationships:
\[
w_t = f(k_t) - k_tw_t f'(k_t) \equiv w(k_t) \quad \text{and} \quad \rho_t = f'(k_t).
\]

From \( s \in (0,1), \ f(k_t) \) and \( w(k_t) \) are strictly concave functions of \( k_t \) and equivalently:
$w'(k_i) > 0 > w''(k_i)$ and $f'(k_i) > 0 > f''(k_i)$. 

For simplicity, capital stock is assumed to depreciate fully in one period. Each agent is endowed with one unit of labor in the first period, which is supplied inelastically to the final goods sector.

There are potentially two assets in the economy, capital and money. It is assumed that $q$ units of final good are required to build one firm. Alternatively, each agent has to access the indivisible investment technology such that $q$ units of final good must be invested in period $t$ to set up one firm in period $t+1$. We simply treat money as an additional asset that can be held by any agent and it plays no special role in transactions. Individuals other than the aged of period one have no endowment of capital and money and in contrast, the initial aged at time one are endowed with the initial capital stock $k_i > 0$ and money supply $M_0 > 0$. The government fixes the money growth rate at the value $\mu$:

$$M_t - M_{t-1} = \mu \cdot M_{t-1}, \quad t \geq 1,$$

where $M_t$ represents the nominal money balances held by young agents at the end of period $t$. The newly created money is used to finance a sequence of real per capita government expenditures $\{g_t\}$, which is endogenously adjusted to satisfy the government budget.

Because government expenditure plays no role in the analysis, we say nothing more about it in what follows.$^2$

2-2. Credit market

The structure of young agents’ preferences is that they care only about old age consumption and thus all young period income is saved. Therefore, their saving rate is set to unity. Noting $N_i = 1$, therefore, wage income $w_i$ is also equal to the level of wealth held by young agents at the end of period $t$. They allocate their income to finance their consumption in period $t+1$ and have two options. Let us define the price level of consumption goods in this period as $P_t$. Noting the definition of $m_i \equiv M_i / P_t$, first, they lend the level of wealth, equal to $w_i - m_i$, in the competitive credit market, which earns a gross return equal to $r_{t+1}$ per unit. In this case, their second-period
consumption becomes \( r_{t+1}(w_t - m_t) + (P_t / P_{t+1})m_t \). Secondly, they start an investment project.

However, we assume that this young period income is not enough to run an investment project.

\[ A.1: \quad q > w(k_t) \quad \text{for all relevant values of} \quad k_t. \]

Thus, if they attempt to start an investment project, they need to obtain external funding equal to \( q - w(k_t) \) in the competitive credit market.

Let us derive the condition for young agents to start an investment project. If they start the project, the second period consumption is equal to \( \rho_{t+1} \cdot q - r_{t+1}[q - w(k_t)] \). This must be greater than or equal to \( r_{t+1}(w_t - m_t) + (P_t / P_{t+1})m_t \), that is the second period consumption, when they hold the amount of real money balances equal to \( m_t \) and lend the residual wealth. If money is valued and loans to capital producers are made at all dates, the returns on these two must be equalized in equilibrium:

\[ r_{t+1} = P_t / P_{t+1}. \]  

Considering (1) and (3), the above condition can be expressed as:

\[ f'(k_{t+1}) \geq P_t / P_{t+1}. \]  

When (4) holds, young agents are willing to borrow and to start the project. We call (4) the profitability constraint.

The credit market is imperfect in the sense that one cannot always borrow any amount at the equilibrium rate \( r_{t+1} \). The borrowing limit exists because the borrower can pledge only up to a fraction of the project revenue for the repayment. More specifically, the borrower would not be able to credibly commit to repay more than \( \lambda \rho_{t+1} q \), where \( 0 < \lambda < 1 \). The agent can start the project, only if \( \lambda \rho_{t+1} q \) is larger than \( r_{t+1}[q - w(k_t)] \). Noting (1) and (3), this relation can be specified as:

\[ \lambda f'(k_{t+1}) \cdot q \geq (P_t / P_{t+1})[q - w(k_t)]. \]

We call (5) the borrowing constraint. Young agents in period \( t \) can start the project only when both (4) and (5) are satisfied. The parameter \( \lambda \) can be interpreted as the degree of credit market friction. The higher is \( \lambda \), the less imperfect is the credit market and, equivalently, the stage of financial market development is higher. If \( \lambda \) were one, the borrowing constraint would never be binding. If it were zero, the agent would never be able to borrow and hence would have to self-finance their project entirely. We consider the whole range of intermediate cases between these two extremes.
If (4) holds as a strict inequality, borrowers would like to borrow an arbitrarily large amount; in this case the borrowing constraint (5) holds as an equality and determines the equilibrium loan. Credit rationing exists when (5) holds as an equality. By contrast, if (4) holds as an equality, there is no difference between the two options facing young agents. In that case, (5) holds as an inequality. Except in exceptional cases, only one of the two equations (4) and (5) will hold as an equality.

Let us define the following.

**Definition 1:** $k_λ$ is the capital–labor ratio such that we have $w(k_λ) = (1 - λ)q$.

The following can be described.

Lemma 1: If the capital stock $t_k$ is larger [smaller] than $k_λ$, the profitability constraint, (4) [the borrowing constraint, (5)] holds as an equality and, in contrast, (5) [(4)] holds as a strict inequality.

The intuition behind the above is very straightforward. If $t_k > (<) k_λ$, the necessity of external finance (the profitability of private capital determined from (5)) is low (high) and thus the borrowing constraint (the profitability constraint) is satisfied in the equilibrium without (with) credit rationing.

Finally, let us derive the equilibrium condition for the funds. In equilibrium, the sources and uses of funds must be equated. If $σ_t$ denotes the proportion of young agents who can access the investment project at $t$, the uses of funds are $σ_t q$ plus real balances; that is $σ_t q + m_t$. The sources of funds are simply per capita saving, that is, $tw(k_t)$. Noting $w(k_t) = (1 - λ)q$, therefore:

$$k_{t+1} = w(k_t) - m_t$$

must hold in equilibrium.

**3. Local dynamics**

3-1. Equilibrium without credit rationing

This subsection focuses on the competitive equilibrium free from credit rationing. In this regime, the nonnegative sequences $(k_t, m_t)$ satisfy (4) and (6) as an equality and (5) as a strict inequality for each $t = 1, 2 ⋅ ⋅ ⋅$, given initial condition $k_1$. Noting $P_t/P_{t+1} = (m_{t+1}/m_t)/(1 + μ)$, it is easy to show that the perfect foresight equilibrium is a sequence of $(k_t, m_t)$ that satisfies: 


In the steady-state equilibrium, the capital–labor ratio and real balances are constant over time. Then, we can derive the following set of the relationships (* denotes the steady-state value):

\[ f'(k_{t+1}) = \frac{(m_{t+1}/m_t)}{1 + \mu}, \]  

(4)′

\[ \lambda f'(k_{t+1}) \cdot q > \frac{(m_{t+1}/m_t) \cdot [q - w(k_t)]}{1 + \mu}, \]  

(5)′

\[ k_{t+1} = w(k_t) - m_t. \]  

(6)

From (7-1), the positive steady-state value of * is uniquely determined. An increase in the money growth rate \( \mu \) reduces the steady-state return on money. For money and the other asset to be held simultaneously, the return on loans must decrease. Consequently, the steady-state value of per capita capital stock is positively related to the money growth rate. Let us derive the condition for the existence of a positive value of *.

**Definition 2:** \( k_U \) is defined as the capital–labor ratio that satisfies \( w(k_U) = k_U \) and \( k_U \) is the strictly positive steady state of \( w(k) \). \( \mu_{PU} \) is given by solving the equation

\[ f'(k_U) = 1/(1 + \mu_{PU}). \]  

(See fig.1)

Noting (1-2) and fig. 1, the existence of the steady-state equilibrium can be summarized as follows.

**Lemma 2:** \( k^* < k_U \) is satisfied, if \( \mu < \mu_{PU} \). Then, (7-3) implies that a positive value of * can be obtained.

**Proof:** See (7-3), fig. 1 and definition 2.

In addition, (7-2) must hold with inequality. Equation (7-2) is ignored here, because section 4 examines whether (7-2) is satisfied at and near the steady state.

Next, we investigate local stability near the steady state. For the purpose, we linearize the equations (4) and (6) about the steady-state level \( (k^*, m^*) \) and can obtain:
\[
\begin{bmatrix}
 k_{t+1} - k^* \\
m_{t+1} - m^*
\end{bmatrix} = \begin{bmatrix}
w'(k^*) & -1 \\
(1 + \mu)m^* f''(k^*) & 1 - (1 + \mu)m^* f''(k^*)
\end{bmatrix}\begin{bmatrix}
k_t - k^* \\
m_t - m^*
\end{bmatrix}.
\] (8)

The determinant of this Jacobian \( D \) is the product of the two eigenvalues and the trace \( Tr. \) is the sum of the eigenvalues. From (1-2), we can compute:

\[
D = w'(k^*) > 0,
\]

(9-1)

\[
Tr. = w'(k^*) + 1 - (1 + \mu)m^* f''(k^*) > 0.
\]

(9-2)

Using (9-1) and (9-2), we obtain the relationship:

\[
Tr. - D - 1 = -(1 + \mu)m^* f''(k^*) > 0.
\]

(10)

Considering (9-1) and (10), the stable root in this Jacobian exists in the range \((0,1)\) and the dynamics of this economy describe a saddle point (local determinacy). Capital stock \( k_t \) is the predetermined variable and nominal money supply \( M_t \) evolves exogenously according to (2). Given the variable \( k_t \), the price level \( P_t \) must be appropriately chosen such that the point \((k_t, M_t, P_t)\) lies on the stable manifold. The implicit function theorem guarantees that such a price level can be uniquely determined if the economy is near the steady state. Then, economic variables such as output, real balances and the capital–labor ratio monotonically approach the steady state. If credit is not rationed to producers, no endogenous fluctuation emerges and local stability is guaranteed.

3-2. Credit rationing equilibrium

This subsection assumes that producers are rationed in the credit market. In this case, the borrowing constraint (5) binds, but the profitability constraint (4) holds as an inequality. It is straightforward to show that the perfect foresight equilibrium is a nonnegative sequence \((k_t, m_t)\) such that:

\[
f'(k_{t+1}) > \frac{(m_{t+1}/m_t)}{1 + \mu},
\]

(4)''

\[
\lambda f'(k_{t+1}) \cdot q = \frac{(m_{t+1}/m_t)}{1 + \mu} [q - w(k_t)],
\]

(5)''

\[
k_{t+1} = w(k_t) - m_t.
\]

(6)

We show that unlike the previous case, a multiplicity of steady states can be observed. The stationary solutions can be obtained by solving the equations:
The function \( H(k^*) \) can be depicted as Fig. 1. We define the level of \( k^* \) at which the minimum value of \( H(k^*) \) can be obtained as follows.

**Definition 3:** \( k_{II} \) is obtained by solving \( H'(k_{II}) = 0 \) and equivalently \( f(k_{II}) = q \) (See fig.1)

An increase in the money growth rate reduces the steady-state return on money and thus the return on loans, which loosens the borrowing constraint. When the capital stock increases, the level of internal finance \( w(k^*) \) rises and the marginal product of capital \( f'(k^*) \) decreases. If \( k^* < k_{II} \), the higher level of internal finance fails to compensate for the reduction in the marginal product of capital. Thus, a fall in the rate of return on loans leads to the required increase in the capital stock \( k_1^* \). If \( k^* > k_{II} \), however, the consequences of a change in the level of internal finance dominates the consequence of a change in the marginal product of capital and thus a fall in the rate of return on loans leads to a fall in the steady-state capital stock \( k_2^* \).

As will be evident in section 5, if the three steady states exist for a given rate of monetary growth \( \mu \), we have \( k_1^* < k_2^* < k_3^* < k_0^* \), where \( k_0^* \) is the steady state in absence of credit rationing. Thus, we refer to the largest (smallest) solution to (13-2), \( k_2^* (k_1^*) \) as the middle- (low-) capital-stock steady state.

If we concentrate on the range of values of \( q \) that ensures the positive level of real balances in the middle-capital-stock steady state \( k_2^* \), the following must be assumed.

**A.2:** \( k_U > k_{II} \) and equivalently \( q < q_0 \equiv \left( \frac{1}{1-s} \right) \left[ A(1-s) \right]^{\frac{1}{1-s}} \). (See fig.1)

Noting A.2, we define the following.
Definition 4: $\mu_H$ and $\mu_{CU} (< \mu_H)$ satisfy
$$\lambda q \cdot H(k_H) = \frac{1}{1 + \mu_H}$$
and
$$\lambda q \cdot H(k_U) = \frac{1}{1 + \mu_{CU}},$$
respectively. (See fig.1)

Let us examine how the rate of monetary growth affects the existence of steady states. For the same reason as in section 3-1, this section also does not examine whether or not the inequality (11-1) is satisfied at the steady states and nearby. We can show the following in the case of credit rationing.

Proposition 1: If $\mu < \mu_{CU}$, the steady state with credit rationing is uniquely determined and is given by $k_1^*$. If $\mu_{CU} < \mu < \mu_H$, there are exactly two steady states with credit rationing, $k_1^*$ and $k_2^*$. There is no monetary steady state if $\mu > \mu_H$.

Proof: Noting that $k^* < k_U$ must be satisfied to obtain the positive value of $m^*$, see fig. 1.

Next, we investigate how the capital–labor ratio and real balances fluctuate near the two steady states. Linearizing (5)'' and (6) around the monetary steady states ($k^*, m^*$) yields:

$$\begin{bmatrix} k_{t+1} - k^* \\ m_{t+1} - m^* \end{bmatrix} = B \begin{bmatrix} k_t - k^* \\ m_t - m^* \end{bmatrix},$$

where:

$$B = \begin{bmatrix} w'(k^*) \\ -k^* f''(k^*) m^* + \left( \frac{1}{1 + \mu} \right) \left( \frac{1}{\lambda q} \right) -1 - \frac{f''(k^*)}{f'(k^*)} m^* \end{bmatrix}.$$ (12)

We can readily establish that the determinant ($D$) and trace of $B$ ($Tr.$) are:

$$D = -k^* f''(k^*) - k^* \frac{f''(k^*)}{f'(k^*)} m^* \left( \frac{1}{1 + \mu} \right) \left( \frac{1}{\lambda q} \right) > 0,$$ (13-1)

$$Tr. = -k^* f''(k^*) + 1 - \frac{f''(k^*)}{f'(k^*)} m^* > 0.$$ (13-2)

If both eigenvalues of $B$ are less than one in modulus, the equilibrium is locally indeterminate. Then, if we abandon the hypothesis of perfect foresight and stochastic perturbations are introduced, there are infinitely many stationary sunspot equilibria near the steady state. See Woodford (1986). In that case, all stationary solutions to (12) can be described as a stochastic process of the form:
\[
\begin{bmatrix}
 k_{t+1} - k^* \\
 m_{t+1} - m^*
\end{bmatrix} = B \begin{bmatrix}
 k_t - k^* \\
 m_t - m^*
\end{bmatrix} + \begin{bmatrix}
 0 \\
 e_{t+1}
\end{bmatrix},
\]

where \( e_t \) is a nonfundamental disturbance (sunspots) with \( e_t \sim N(0, \sigma_e) \).

Considering (13), we can describe the following proposition.

**Proposition 2:** The low-capital-stock steady state is necessarily a saddle, while the middle-capital-stock steady state is either a sink or a source, depending upon the rate of monetary growth and the indivisible investment scale. The steady states exhibit a saddle-node bifurcation at \( \mu = \mu_H \).

Proof: See appendix A.

Moreover, the following can be added in relation to the stability of the middle-capital-stock steady state.

**Proposition 3:** Suppose that

\[
\begin{bmatrix}
 s & ss \\
 ss & q
\end{bmatrix} > \begin{bmatrix}
 s & ss \\
 ss & q
\end{bmatrix}^{-1} \begin{bmatrix}
 1 & 1/s \\
 1 & 2/s
\end{bmatrix} \begin{bmatrix}
 A(1-s) \\
 1
\end{bmatrix}^{1/2} \quad \text{is satisfied.}
\]

Then, the middle-capital-stock steady state is necessarily a sink (locally indeterminate) for \( \mu_{\text{CU}} < \mu < \mu_H \).

In contrast, suppose that \( q < \bar{q} \) is satisfied. The steady state is a sink when \( \mu_{\text{CU}} < \mu < \mu_{\text{NS}} \), displays a Neimark-Sacker bifurcation at \( \mu = \mu_{\text{NS}} \) and a source when \( \mu_{\text{NS}} < \mu < \mu_H \).

Proof: See appendix B.

If the scale at which the investment project must be operated is relatively large (i.e., \( q > \bar{q} \)) or the rate of monetary growth is moderately low (i.e., \( \mu_{\text{CU}} < \mu < \mu_{\text{NS}} \) and \( q < \bar{q} \)), local indeterminacy of equilibrium arises and there exist an infinite number of stationary sunspot equilibria around the middle-capital-stock steady state \( k^*_2 \). Thus, agents’ expectation-driven fluctuations emerge near the steady state. If \( q < \bar{q} \), the middle-capital-stock steady state can generate endogenous cycles through a Neimark-Sacker bifurcation, which produces invariant closed curves.

To clarify the intuition behind proposition 3, let us show that local indeterminacy is more likely when the scale of indivisible investment \( q \) is higher. Suppose that deflation is expected to occur, i.e., \( e_{t+1} > 0 \). Noting that the growth rate of nominal money is fixed, this means that the growth rate of real balances is expected to increase. From (5)”, such expectations tighten the borrowing constraint and generate lower levels of capital investment in the next period \( k_{t+1} \).
level of \( k_{t+1} \) is smaller, as \( q \) is higher. Then, (6) implies that the higher level of price in this period \( P_t \) is realized for a given value of \( k_t \) and thus agents’ beliefs about deflation become more self-fulfilling.

4. Monetary policy and the existence of steady states

This section analyzes how the rate of monetary growth determines the existence of steady states with or without credit rationing, depending upon the degree of credit market friction \( \lambda \) and particularly the indivisible investment scale \( q \). We can easily understand the existence of the steady states by examining the intersections of the horizontal line \( 1/(1 + \mu) \) and the downward sloping curve \( f'(k^*) \) or the U shaped curve \( \lambda q \cdot H(k^*) \). Furthermore, we must investigate whether the steady state(s) without (with) credit rationing is (are) larger (smaller) than \( k_\lambda \). See lemma 1. Noting definition 1, let us define the following.

**Definition 5:** \( k_\lambda \) and \( \mu_\lambda \) are obtained by solving the equation \( \lambda q \cdot H(k_\lambda) = f'(k_\lambda) = \frac{1}{1 + \mu_\lambda} \).

(See fig.1.)

From definitions 3 and 5, the next relationship can easily be derived.

Lemma 3: \( k_\lambda < k_H \) is satisfied, if \( 1 > \lambda > s \), while \( k_\lambda > k_H \) is satisfied, if \( 0 < \lambda < s \), where \( s \) is the share of capital stock in relation to production. (See fig.2)

First, we consider the case of \( 1 > \lambda > s \). In this case, credit market frictions are sufficiently small and thus high levels of financial development are achieved. From figs.1 and 3, we can see that the existence of the steady state is extremely simple and the outcome can be summarized as the next proposition and fig.4.

**Proposition 4:** There exists a unique low-capital-stock steady state with credit rationing \( k_1^* \) when \( \mu < \mu_\lambda \). A unique steady state without credit rationing \( k_0^* \) exists when \( \mu_\lambda < \mu < \mu_{PU} \). No monetary steady state exists for \( \mu > \mu_{PU} \).

Proof: See appendix C, where the intuitions behind the results are also described.
When $0 < \lambda < s$, credit market frictions are relatively large and thus relatively low levels of financial development are realized. Unlike the case of $1 > \lambda > s$, fig.2 implies how the existence of the steady states depends upon the scale at which the investment project must be operated. We show the proposition, having defined the following and stated the lemma.

**Definition 6:**

\[ q_2 = \frac{1}{1-s} \left( \frac{\lambda}{s} \right)^{1-s} \left[A(1-s)\right]^{1/s} \quad \text{and} \quad q_2 < q_1 = \frac{1}{1-\lambda} \left[A(1-s)\right]^{1/s} < q_0. \]

Lemma 4: $q_2$ is such that $\mu_{H}$ and $\mu_{PU}$ coincide. $q_1$ is such that $\mu_{CU}$ and $\mu_{PU}$ coincide.

Proof: From definition 6, we can understand $\text{sgn}(\mu_{H} - \mu_{PU}) = \text{sgn}(q - q_2)$ and $\text{sgn}(k_U - k_A) = \text{sgn}(q_1 - q)$. Using $1 + \mu_{PU} = (1-s)/s$ and $1 + \mu_{H} = (q/A)^{1/s} \lambda^{-1} q^{-1}$, we get $\mu_{H} > (\cdot)\mu_{PU}$ for $q > (\cdot)q_2$. Noting definitions 2 and 5, $k_A \geq (\cdot)k_U$ is verified for $q \geq (\cdot)q_1$. Thus, we can see $\mu_{PU} \leq (\cdot)\mu_{A} \leq (\cdot)\mu_{CU}$.

**Proposition 5:**

Case 1. $q_2 > q > 0$: When $\mu < \mu_{A}$, the uniqueness of steady state $k_1^*$ is clarified. If $\mu_{A} < \mu < \mu_{H}$, we can verify the multiplicity of steady states $k_0^*, k_1^*$ and $k_2^*$. If $\mu_{H} < \mu < \mu_{PU}$, the steady state $k_0^*$ is uniquely obtained. No steady state exists for $\mu > \mu_{PU}$. (See fig.5.)

Case 2. $q_1 > q > q_2$: The steady state with credit rationing $k_1^*$ uniquely exists for $\mu < \mu_{A}$. When $\mu_{PU} > \mu > \mu_{A}$, there exist multiple steady states $k_0^*, k_1^*$ and $k_2^*$. If $\mu_{H} > \mu > \mu_{PU}$, we can confirm the multiplicity of steady states with credit rationing $k_1^*$ and $k_2^*$. No steady state exists for $\mu > \mu_{H}$. (See fig.6.)

Case 3. $q_0 > q > q_1$: When $\mu < \mu_{CU}$, the steady state $k_1^*$ can be uniquely obtained. If $\mu_{CU} < \mu < \mu_{H}$, we can obtain the multiplicity of steady states $k_1^*$ and $k_2^*$. There exists no monetary steady state, if $\mu > \mu_{H}$. (See fig.7.)

Proof: See appendix E, where the reasons for the analytical outcomes are explained.

We can summarize the proposition 5 as fig.8.
5. Poverty traps and financial development

Using the consequences in section 4, this section clarifies how financial development affects the likelihood of poverty traps. As described in section 1, poverty traps occur as an inevitable outcome when the economy is trapped in either of the two situations explained below.\(^5\)

**Situation 1:** A low-capital-stock steady state exists, but middle- and high-capital-stock steady states do not exist.

**Situation 2:** Low- and middle-capital-stock steady states exist, but there exist no equilibrium path converging to a middle-capital-stock steady state.

If either of situations 1 or 2 occurs, the economy is inevitably locked into a poverty trap, because it is impossible to experience transition from the low-capital-stock steady state to the middle- or the high-capital-stock steady state.

Let us restrict attention to the range of values of \(q\) as follows (the alternative ranges are discussed in footnotes 7 and 8).

A.3: \[ A(1-s)^{1/(sA)} < q < \bar{q}. \]^\(6\)

From definition 7, we can easily derive the following.

**Lemma 5:** \[ \frac{dq_i}{d\lambda} > 0 \ (i = 1, 2). \] Moreover, \[ q_i \rightarrow q_0 \left( \equiv \frac{1}{1-s} \left[ A(1-s)^{1/(sA)} \right] \right) \ (i = 1, 2), \] as \(\lambda \rightarrow s\), while \(q_2 \rightarrow 0\) and \(q_1 \rightarrow A(1-s)^{1/(sA)}\), as \(\lambda \rightarrow 0\). For a given value of \(q\), A.3 implies that there exists a unique positive pair \((\varepsilon_0, \varepsilon_1)\) such that \(s - \varepsilon_1 < \lambda < s \iff 0 < q < q_2\), \(\varepsilon_0 < \lambda < s - \varepsilon_1 \iff q_2 < q < q_1\) and \(0 < \lambda < \varepsilon_0 \iff q_1 < q < q_0\).

Fig. 9 characterizes the degrees of \(\lambda\) to obtain \(\mu_{PU}\) equated with \(\mu_H, \mu_{NS}\) and \(\mu_{CU}\), if \(\lambda\) is lower in the interval \((0, s - \varepsilon_1)\). As for the definitions of \(\lambda_{NS}\) and \(k_{XS}\), see the note in fig.9.

Using figs.1, 2 and 9, we can illustrate fig.10-1. Fig. 10-1 shows how the degree of \(\lambda\) affects the appearances of the situations 1 and 2 when \(\lambda\) is lower in the range \((0, 1)\), depending on the rate of monetary growth. For the proof of fig.10-1, the following can be explained. When \(1 > \lambda > \varepsilon_0\) \((\varepsilon_0 > \lambda > 0)\), situation 1 arises for the range of money growth rates \(\mu < \mu_2\) \((\mu_{CU})\).
When $1 > \lambda > s$ ($s > \lambda > s - \varepsilon_1$), the high-capital-stock steady state $k_0^*$ or steady growth process is necessarily realized for $\mu_{PU} > \mu > \mu_\lambda (\mu_H)$. However, a decline in $\lambda$ raises the values of $\mu_\lambda, \mu_{CU}$ and $\mu_H$, while the level of $\mu_{PU}$ is unaffected by changes in $\lambda$. If $s - \varepsilon_1 > \lambda > \lambda_{NS}$ [$\lambda_{NS} > \lambda > 0$], situation 2 emerges for the interval $(\mu_{PU}, \mu_H)$ $[(\mu_{NS}, \mu_H)]$.

Consider that the degree of financial development is lower in the region $(0, \lambda_{NS})$. See fig. 11. Then it is shown that situation 2 occurs over a larger interval of money growth rates if the share of physical investment to total saving is sufficiently high at $k_2^* = k_{NS}$. Under this condition, appendix E clarifies $\frac{\partial}{\partial \lambda} (\mu_H - \mu_{NS}) < 0$ for $\lambda_{NS} > \lambda > 0$.

Fig. 10-1 indicates in which regions there would be poverty traps of ‘situation 1’ or ‘situation 2’ for the money growth rate and financial development and clarifies the following. Because financial markets are less developed, situations 1 and 2 (steady growth process) emerge(s) over a larger (a smaller) range of money growth rates. Therefore, we can conclude that a poverty trap is more likely, the less developed are financial markets. This implies a positive link between financial development and per capita income. 7 8

6. Global dynamics

This section considers the global dynamics to enhance the representation of local dynamics in section 5. The analysis of global dynamics can be regarded as contributing to the understanding of the results regarding the local dynamics. Because we focus on the Cobb–Douglas technology, the global dynamics of capital and real balances can be correctly illustrated as phase diagrams in the $(k, m)$ plane. Using the phase diagrams, let us demonstrate that the economy is encased in the poverty trap as a result of the low stage of financial development, even if the economic variables are far away from the steady states.

Considering (6), we can obtain:

$$k_{t+1} - k_t = w(k_t) - k_t - m_t.$$  \hspace{1cm} (14-1)

From (14-1), the following can be readily understood:

$$k_{t+1} \gg k_t \Leftrightarrow w(k_t) - k_t \gg m_t.$$  \hspace{1cm} (14-2)

As for (4)', which determines the global dynamics of real balances in the equilibrium free from credit rationing, we can see:

15
\[ \frac{m_{t+1}}{m_t} \gtrsim 1 \]
\[ \Leftrightarrow (1 + \mu) f'(k_{t+1}) \gtrsim 1 \]
\[ \Leftrightarrow k_{t+1} \gtrsim k_0^* \]  \hspace{1cm} (15-1)
\[ \Leftrightarrow w(k_t) - k_0^* \gtrsim m_t. \]  \hspace{1cm} (15-2)

(15-1) can be readily derived from \((1 + \mu) f'(k_0^*) = 1\) or (7-1), while substituting (6) into (15-1) yields (15-2).

Regarding (5)'', which governs the global motions of real balances in the equilibrium with credit rationing, we can verify:

\[ \frac{m_{t+1}}{m_t} \gtrsim 1 \]
\[ \Leftrightarrow (1 + \mu) \lambda q \cdot \left[ sA k_{t+1}^{r-1} \right] \gtrsim 1 \]  \hspace{1cm} (16-1)
\[ \Leftrightarrow m_t \gtrsim w(k_t) - \left[ \frac{(1 + \mu) \lambda qsA}{q - w(k_t)} \right]^{\frac{1}{r-1}}. \]  \hspace{1cm} (16-2)

Substituting (6) into (16-1) and arranging the equation provide (16-2). See appendix F in regard to the feature of the steady locus of real balances in this equilibrium.

Let us depict the phase diagrams in the \((k_t, m_t)\) plane. As shown in lemma 1, the global dynamics are governed by (15) [(16)] and (14), when the level of capital stock \(k_t\) is higher [smaller] than \(k_\lambda\). Then, the borrowing constraint (5)'' [the profitability constraint (4)'] is satisfied and thus the equilibrium without [with] credit rationing arises. To save space, we consider only the global dynamics in the relatively low stages of financial development where \(\lambda_{NS} < \lambda < s - \epsilon_1\).

Considering (14)--(16), then, the phase diagrams can be shown as figs. 12-14, depending on the rate of monetary growth.

Fig. 12 focuses on the range of the rate of monetary growth \(\mu < \mu_\lambda\) and considers the global dynamics in the case of situation 1. Noting \(k_0^* < k_\lambda < k_2^*\) in this figure, the profitability (the borrowing) constraint is not satisfied in the middle- (the high-) capital-stock steady state and thus only the low-capital-stock steady state exists. Even if the initial level of per capita GDP largely
surpasses the critical value $k_\lambda$, the economy necessarily converges to the low-capital-stock steady state and is mired in the low level of aggregate activity.

Fig. 13 concentrates on the range of $\mu_{NS} < \mu < \mu_{PU}$. From $k_1^* < k_2^* < k_\lambda < k_0^*$ in this figure, there coexist the three steady states. As there exists no equilibrium path leading to the middle-capital-stock steady state, the economy converges to either the high- or the low-capital-stock steady state, depending on the chosen initial value of real balances. Unlike the fig. 12, it is ambiguous which the economy is locked in the development traps or enjoys the steady growth process.\(^9\)

When $\mu_{PU} < \mu < \mu_H$, fig. 14 shows the appearance of situation 2. This figure illustrates that the steady-state level of capital stock $k_0^*$ is not small enough to give a positive value of real balances and thus, only the low- and the middle-capital-stock steady states coexist. Because no convergent path to the middle-capital-stock steady state exists, the economic variables necessarily approach the low-capital-stock steady state. Even if the initial position of the economy is sufficiently near the middle-capital-stock steady state, the economy is inevitably confined to the development trap, as shown in fig. 14.

7. Conclusions

Using Diamond’s overlapping generations model with credit market frictions, this paper explores how monetary policy and the degree of financial development influence the occurrence of local indeterminacy, local bifurcations, multiple steady states and, particularly, poverty traps.

The relationship between the degree of financial development and the occurrence of poverty traps can be described as follows. Suppose that the rate of monetary growth is lower than some critical value. Then, the middle- and high-capital-stock steady states disappear and only the low-capital-stock steady state exists. The lower the degree of financial development, the higher the critical rate of monetary growth. By contrast, a multiplicity of steady states can be verified for moderately high rates of monetary growth. However, with financial development sufficiently low, the high-capital-stock steady state disappears and the middle-capital-stock steady state loses its stability for a wider interval of money growth rates.

Therefore, a poverty trap emerges as an inevitable outcome over a larger range of money growth rates, in proportion to the degree of underdevelopment of the financial system. A decline in financial development raises the likelihood that the economy will be locked in the low-capital-stock steady state.

Finally, let us summarize how the analytical consequences of monetary policy can be stated, depending on the stage of development of financial markets. If the level of financial development is
above the capital share to total income, the economy is globally determinate in the sense that the money growth rate determines which of the steady states will be realized. Moreover, the steady state is locally determinate because no endogenous fluctuation emerges nearby. If the level of financial development is below the capital share to total income, the economy can be globally indeterminate in that we do not know which of the steady states is obtained for some interval of money growth rates. Moreover, the middle-capital-stock steady state can be locally indeterminate with an infinite number of stationary sunspot equilibria and displays local endogenous cycles through Neimark-Sacker bifurcation at some specific rate of monetary growth. Therefore, the stage of development of the financial system is also important in considering macroeconomic instability.

Appendix A (Proof of proposition 2)

From (13-1) and (13-2), we can derive:

\[
D - Tr. + 1 = -m^* \frac{f''(k^*)}{f'(k^*)} \left[-1 + \left(\frac{1}{1 + \mu}\right)\left(\frac{1}{\lambda q}\right) k^*\right].
\] (A-1)

To verify the sign of (A-1), let us substitute (11-2) into:

\[
H'(k^*) = \frac{1}{[q - w(k^*)]^2} \left[f''(k^*)\{q - f(k^*)\}\right].
\]

Considering fig. 1, we obtain:

\[
H'(k^*) = -\frac{f''(k^*)}{f'(k^*)} \left(\frac{1}{1 + \mu}\right) \left[1 + \left(\frac{1}{1 + \mu}\right)\left(\frac{1}{\lambda q}\right) k^*\right] > 0 \ (< 0) \text{ at } k^* = k_z^*(k_i^*).
\] (A-2)

From (13-1) and (13-2), both \( Tr. \) and \( D \) are positive. Thus, (A-1) and (A-2) mean that the low-capital-stock steady state is a saddle (locally determinate) and, in contrast, the high-capital-stock steady state \( k_z^* \) is a sink (locally indeterminate) or a source (locally divergent).

Appendix B (Proof of proposition 3)

Considering appendix A, the high-capital-stock steady state \( k_z^* \) is a sink when \( 0 < D < 1 \) and a source when \( D > 1 \). As \( \mu \to \mu_{CU} \), fig. 1 shows \( k_z^* \to k_U \) and thus \( m_z^* \to 0 \). From (1-2), (13-1) and \( k_U = w(k_U) \), then, we can see that:

\[
D \to -k_U \cdot f''(k_U) \equiv w'(k_U) < 1.
\] (B-1)

Note that Cobb–Douglas technology is utilized in the present paper. By using (11-2) and (11-3), (13-1) can be rewritten as:
\[ D = As(1-s)(k^*)^{s-1}, \quad \frac{q-k^*}{q-A(1-s)(k^*)^s}. \] (B-2)

Considering A.1 and \( D > 0 \), we easily confirm that:

\[ g(k^*) < (>)q \iff D < (>)1, \]
\[ g(k^*) = A(1-s)^2(k^*)^s + Aqs(1-s)(k^*)^{s-1}. \] (B-3)

From \( \sigma^* q = k^* \) and \( 0 < \sigma^* < 1 \), we verify \( q > k^* \). Noting this relationship, we differentiate \( g(k^*) \) with respect to \( k_2^* \) and get:

\[ g'(k^*) = A(1-s)^2(k^*)^{s-2}(k^*-q) < 0. \] (B-4)

Noting \( k_H = (q/A)^{1/s} \), we see:

\[ g(k_H) < (>)q \iff q > (q)q = \left( \frac{1}{1-s} \right)^{1/s} \left[ A(1-s) \right]^{1-s}. \] (B-5)

As indicated in fig.1, \( k_H < k_U \) should be noted. From (B-1), (B-3), (B-4) and (B-5), thus, the following can be stated. When \( q > \bar{q} \), the high-capital-stock steady state \( k_2^* \) is necessarily a sink for the feasible range of money growth rates \( \mu_{CU} < \mu < \mu_H \). However, suppose that \( q < \bar{q} \) is satisfied. Here, let us briefly explain the condition for the emergence of Neimark-Sacker bifurcation that generates invariant closed curves around the steady state. Noting that the mapping has a smooth family of steady states \( (k_2^*(\mu), m_2^*(\mu)) \), the condition is that the two eigenvalues are complex conjugates \( \lambda(\mu), \ \bar{\lambda}(\mu) \) and cross the unit circle with nonzero speed, when the money growth rate \( \mu \) varies as a bifurcation parameter. Let us define the following.

**Definition 7:** \( \mu_{NS} \) is defined as the money growth rate such that \( \lambda(\mu_{NS}) = 1 \) but \( \lambda'(\mu_{NS}) \neq 1 \) for \( j = 1, 2, 3, 4^{10} \) and \( \frac{d}{d\mu}(\lambda(\mu_{NS})) \neq 0 \). In this model, \( \mu_{NS} \) can be obtained by solving the equation \( g(k_2^*) = q \).

From (B-1), (B-3) and fig. 1, the following is readily derived. Noting \( dk_2^*/d\mu < 0 \), the steady state is a sink for \( \mu_{CU} < \mu < \mu_{NS} \), undergoes a Neimark-Sacker bifurcation at \( \mu = \mu_{NS} \) and is a source for \( \mu_{NS} < \mu < \mu_H \).
Appendix C (Proof of proposition 4)

When $\mu < \mu_\lambda$, fig. 3 implies that (7-2) is not satisfied in the steady state without credit rationing $k_0^*$:

$$f'(k_0^*) = \frac{1}{1 + \mu}$$ \hspace{1cm} (C-1)

$$\lambda q \cdot \frac{f'(k_0^*)}{[q - w(k_0^*)]} < \frac{1}{1 + \mu}.$$ \hspace{1cm} (C-2)

Combining (C-1) and (C-2) yields $w(k_0^*) < (1 - \lambda)q$ and thus $k_0^* < k_\lambda$, which is not consistent with the existence of the equilibrium without credit rationing. See lemma 1. From fig. 3, (11-1) is (not) satisfied in the low- (the high-) capital-stock steady state with credit rationing $k_1^*$ ($k_2^*$):

$$f'(k_1^*) > \frac{1}{1 + \mu},$$ \hspace{1cm} (C-3)

$$f'(k_2^*) < \frac{1}{1 + \mu},$$ \hspace{1cm} (C-4)

$$\lambda q \cdot \frac{f'(k_i^*)}{[q - w(k_i^*)]} = \frac{1}{1 + \mu}, \hspace{0.5cm} (i = 1, 2).$$ \hspace{1cm} (C-5)

Combining (C-3) [(C-4)] with (C-5), we can obtain $w(k_1^*) < (1 - \lambda)q$ [\(w(k_2^*) > (1 - \lambda)q\)], which is compatible (incompatible) with the existence of the equilibrium with credit rationing. See lemma 1. Thus, the uniqueness of the steady state $k_1^*$ is confirmed for this range of money growth rates.

If $\mu_\lambda < \mu < \mu_H$, fig. 3 means that (11-1) is not satisfied in the two steady states with credit rationing $k_1^*$ and $k_2^*$, i.e., $w(k_i^*) > (1 - \lambda)q$ ($i = 1, 2$), but (7-2) is satisfied in the steady state without credit rationing $k_0^*$, i.e., $w(k_0^*) > (1 - \lambda)q$. Thus, the steady state $k_0^*$ is uniquely obtained.

It is easy to see that the steady state $k_0^*$ is determined uniquely for $\mu_H < \mu < \mu_{PU}$. Thus, the uniqueness of steady state $k_0^*$ is verified for $\mu_\lambda < \mu < \mu_{PU}$.

Appendix D (Proof of proposition 5)
Case 1 \((q_2 > q > 0)\):

Fig. 5 shows that when \(\mu < \mu_\lambda\), (7-2) is not satisfied in the steady state \(k_0^*\). Thus, (D-1) and (D-2) can be also obtained in this case. We can get \(w(k_0^*) < (1-\lambda)q\) incompatible with the existence of the equilibrium without credit rationing. From Fig. 5, (D-3)-(D-5) can be also obtained. We can also get \(w(k_1^*) < (1-\lambda)q\) and \(w(k_2^*) > (1-\lambda)q\). Thus, the uniqueness of the steady state with credit rationing \(k_1^*\) is confirmed.

If \(\mu_{ii} > \mu > \mu_\lambda\), (11-1) is satisfied in the two steady states \(k_1^*\) and \(k_2^*\):

\[
f'(k_i^*) > \frac{1}{1+\mu}, \quad (D-1)
\]

\[
\lambda q \cdot \frac{f'(k_i^*)}{[q - w(k_i^*)]} = \frac{1}{1+\mu}, \quad (i = 1, 2). \quad (D-2)
\]

Combining (D-1) and (D-2) generates \(w(k_i^*) < (1-\lambda)q\) consistent with the existence of the steady state with credit rationing. As (7-2) is satisfied in the steady state \(k_0^*\), we can obtain

\[
f'(k_0^*) = \frac{1}{1+\mu}, \quad (D-3)
\]

\[
\lambda q \cdot \frac{f'(k_0^*)}{[q - w(k_0^*)]} > \frac{1}{1+\mu}. \quad (D-4)
\]

From (D-3) and (D-4), we can get \(w(k_0^*) > (1-\lambda)q\) compatible with the existence of the steady state without credit rationing. Therefore, steady states with and without credit rationing coexist.

If \(\mu_{ii} > \mu > \mu_{ii}\), the uniqueness of the steady state without credit rationing \(k_0^*\) is easily verified, while no steady state exists if \(\mu > \mu_{PV}\).

Case 2 \((q_1 > q > q_2)\):

Considering the case 1 and Fig. 6, a unique steady state with credit rationing \(k_1^*\) exists for \(\mu < \mu_\lambda\). When \(\mu_{PV} > \mu > \mu_\lambda\), we can confirm the co-existence of the two kinds of steady states.

See the interval \(\mu_{ii} > \mu > \mu_\lambda\) in the case 1. If \(\mu_{ii} > \mu > \mu_{PV}\), \(w(k^*) < (1-\lambda)q\) is satisfied in the two steady states \(k_1^*\) and \(k_2^*\), but the steady state capital stock \(k_0^*\) is not small enough for the steady state value of real balances \(m_0^*\) to be positive. Therefore, a multiplicity of steady
states with credit rationing $k_1^*$ and $k_2^*$ is confirmed. If $\mu > \mu_H$, there exists no monetary steady state equilibrium.

Case 3 ($q_0 > q > q_1$):

When $\mu < \mu_{CU}$, the steady state $k_1^*$ can be uniquely obtained. See Fig. 7 and the range $\mu < \mu_\lambda$ in the case 1.\footnote{When $\mu_H > \mu > \mu_{CU}$, the arguments for $\mu_H > \mu > \mu_{PU}$ in the case 2 are true of this interval. Thus, we can verify a multiplicity of steady states with credit rationing.} When $\mu$ increases through $\mu_H$, the two steady states with credit rationing coalesce and disappear. To put it another way, a saddle-node bifurcation occurs at $\mu = \mu_H$ and there exists no monetary steady state, if $\mu > \mu_H$.

Appendix E (Proof of $\frac{\partial}{\partial \lambda} (\mu_H - \mu_{NS}) < 0$ for $\lambda_{NS} > \lambda > 0$ in fig. 10)

Let us prove that $\mu_H - \mu_{NS}$ is negatively related to $\lambda$ for the interval $0 < \lambda < \lambda_{NS}$. Using the determinant equal to one ($D = 1$), we obtain:

$$1 + \mu_{NS} = \frac{(1-s)[A(1-s)k_{NS}^s - k_{NS}]}{1 - As(1-s)k_{NS}^{s-1}} \left( \frac{1}{\lambda q} \right).$$

(E-1)

From $g(k_{NS}) = q$, the denominator on the right hand in (E-1) can be expressed as:

$$1 - As(1-s)k_{NS}^{s-1} = \frac{A(1-s)^2 k_{NS}^s}{q}.$$

(E-2)

Substituting (E-2) into (E-1) yields:

$$1 + \mu_{NS} = \frac{[A(1-s)k_{NS}^s - k_{NS}]}{A(1-s)k_{NS}^s} \left( \frac{1}{\lambda} \right)$$

$$= \frac{m_{NS}}{w(k_{NS})} \left( \frac{1}{\lambda} \right).$$

(E-3)

Considering (E-3) and $1 + \mu_H = \left( \frac{q}{A} \right)^\frac{1}{1-\varepsilon} \left( \frac{1}{\lambda q} \right)$, $\frac{\partial}{\partial \lambda} (\mu_H - \mu_{NS}) < 0$ is satisfied, if

$$q^{\frac{1}{1-\varepsilon}} > \frac{m_{NS}}{w(k_{NS})} \cdot A^{\frac{1}{1-\varepsilon}}.$$

(E-4)
Noting that $m_{NS} \to 0$, as $k_{NS} \to k_U$, (E-4) is satisfied.

Appendix F (The feature of $m_i = m_i(k_i)$)

Let us consider the feature of the steady locus of real balances in the equilibrium with credit rationing $m_{t+1} = m_i$;

$m_i = m_i(k_i),$

where

$$m_i(k_i) \equiv w(k_i) - \left( \frac{(1 + \mu) \lambda q s A}{q - w(k_i)} \right)^{\frac{1}{1-\gamma}}.$$  \hspace{1cm} (F-1)

The relationships are easily derived;

$$\lim_{k_i \to 0} m_i(k_i) = -\left[ (1 + \mu) \lambda s A \right]^{\frac{1}{1-\gamma}} < 0,$$  \hspace{1cm} (F-2)

$$\lim_{k_i \to k_{crit}} m_i(k_i) = -\infty$$  \hspace{1cm} (F-3)

and

$$m_i'(k_i) = 0 \text{ at } k_i = \tilde{k}_i.$$  \hspace{1cm} (F-4)

$\tilde{k}_i$ is the unique value of $k_i$ satisfying

$$(1-s)\left[ (1 + \mu) \lambda q s A \right]^{\frac{1}{1-\gamma}} = \left[ q - w(k_i) \right]^{\frac{2-\gamma}{1-\gamma}}.$$  

Considering (F-2) - (F-4), we can illustrate the steady locus of real balances as in figs 12-14.

Acknowledgement

I would like to thank two referees and an associate editor for making a number of suggestions and criticisms that have helped improve my paper. I also thank Tatsuro Iwaisako, Keiichi Hori, Hiroshi Kinokuni, Junko Doi, Takashi Ono, Atsushi Miyake, and seminar participants at Ritsumeikan University and Kansai University for their helpful discussions. The financial support of the Grant-in-Aid for Scientific Research (No. 21530177) is greatly appreciated.
Footnote

1 In Gokan (2006, 2008), the importance of government fiscal policy was emphasized in analyzing whether local indeterminacy and endogenous cycles through Neimark-Sacker and/or flip-bifurcations can emerge.

2 Even if we assume a helicopter-drop of money and the government distributes the newly created money equally among the two types of agents, lenders and borrowers, the equations obtained are the same as (4) and (5).

3 Needless to say, we consider the case where the borrowing constraint (5) binds.

4 It is easily proved that \( q_2 < q_1 \) is satisfied only if \( \lambda \neq s \).

5 We denote the steady state without credit rationing as the high-capital-stock steady state.

6 If \( \left( \frac{1}{1-s} \right) \left( \frac{1}{2-s} \right)^{\frac{1}{1-s}} > 1 \), \( \bar{q} > \left[ A(1-s) \right]^{\frac{1}{1-s}} \) can be verified. Clearly, this inequality is always satisfied for any values of \( s \in (0,1) \).

7 If we consider the range \( 0 < q < \left[ A(1-s) \right]^{\frac{1}{1-s}} \), the region of \( 0 < \lambda < \varepsilon_0 \) in fig. 10 is eliminated and \( \varepsilon_0 \) in fig. 10 is replaced by 0. Thus, the implications derived above are not changed in this alternative case.

8 When we think of the range of \( q_0 > q > \bar{q} \), the middle-capital-stock steady state is always a sink (locally indeterminate), only if the existence of the middle-capital-stock steady state exist. Thus, situation 2 does not emerge, but a decline in the degree of financial development raises the likelihood of situation 1, as in fig. 10-1. See fig. 10-2. The positive impacts on aggregate activity of the development of financial markets are weaker, but we can also observe a positive connection between them in this case.

9 There exist an infinite number of equilibrium path leading to the middle capital-stock-steady state for \( \mu_\lambda < \mu < \mu_{NS} \). As we can easily imagine the feature of phase diagram in this case from fig. 13, this case is omitted from Section 6.
\[ \lambda^4(\mu_{NS}) \neq 1 \] rules out the cases of \( \lambda(\mu_{NS}) = \pm 1, \pm i \) and \( \lambda^3(\mu_{NS}) = 1 \) eliminates the case of
\[ \lambda(\mu_{NS}) = \frac{-1 \pm \sqrt{3}}{2} . \] Thus, it is no problem, even if \( j = 3, 4 \) is written in definition 8.

11 For \( \mu < \mu_{CU} \), we can see \( w(k_2^*) > (1 - \lambda)q \) and the capital stock \( k_2^* \) is not small enough to verify the positive value of \( m_2^* \).

12 When \( \mu_{CU} > \mu > \mu_{PU} \), we can obtain \( w(k_0^*) < (1 - \lambda)q \), but unlike the interval \( \mu_{II} > \mu > \mu_{L} \) in the case 1, the capital stock \( k_0^* \) is so large that the positive value of \( m_0^* \) cannot be verified.
Fig. 1: Definitions

Note: $k_{\text{INF}}$ is the capita-labor ratio satisfying $q = w(k_{\text{INF}})$. 
Fig. 2: The positions of \( \lambda q \cdot H(k^*) \) for the sizes of \( q \)

Note:

The curve \( - \) is the case of \( 1 > \lambda > s \).
The curve \( \textcolor{blue}{-} \) denotes the case of \( q_2 > q > 0 \) \( (s > \lambda > s - \varepsilon_1) \).
The curve \( \textcolor{green}{--} \) shows the case of \( q_1 > q > q_2 \) \( (s - \varepsilon_1 > \lambda > \varepsilon_0) \).
The curve \( \textcolor{red}{-} \) represents the case of \( q_0 > q > q_1 \) \( (\varepsilon_0 > \lambda > 0) \).
Note:
Noting that $\mu_\lambda$ is positively related to $q$, this figure shows that the steady state $k_1^*$ exists for $\mu < \mu_\lambda$ and the existence of $k_0^*$ is verified for $\mu_\lambda < \mu < \mu_{PU}$.
Fig. 5: \( q_2 > q > 0 \) \((s > \lambda > s - \varepsilon_i)\)

Fig. 6: \( q_1 > q > q_2 \) \((s - \varepsilon_i > \lambda > \varepsilon_0)\)
Fig. 7: $q_0 > q > q_1$ ($\epsilon_0 > \lambda > 0$)
Fig. 8: The existence of steady states for values of \( q \) and \( \mu \), when \( 0 < \lambda < s \)

Note:
For example, this figure shows the following when \( q_1 > q > q_2 \). The steady state \( k_1^* \) uniquely exists for \( \mu < \mu_3 \). When \( \mu_{PU} > \mu > \mu_3 \), there exist multiple steady states \( k_0^*, k_1^*, \text{ and } k_2^* \). If \( \mu_H > \mu > \mu_{PU} \), we can confirm the multiplicity of steady states \( k_1^* \text{ and } k_2^* \). No steady state exists for \( \mu > \mu_H \).
Fig. 9: The critical values of $\lambda$

Note:

$\lambda_{NS}$ is defined as the value of $\lambda$ in the interval $(\varepsilon_0, s - \varepsilon_1)$ such that we have $\mu_{NS} = \mu_{PU}$. ($k_{NS}$ is determined by the equation $g(k^*) = q$. See eq. B.3.)

The curve $\lambda = \lambda_{NS}$ shows that $\mu_H = \mu_{\lambda}$ is satisfied, when $\lambda = \lambda_{NS}$.

The curve $\lambda = s - \varepsilon_1$ shows that $\mu_{PU}$ is satisfied, when $\lambda = s - \varepsilon_1$.

The curve $\lambda = \varepsilon_0$ shows that $\mu_{CU} = \mu_{PU}$ is satisfied, when $\lambda = \varepsilon_0$. 

The curve $\lambda = \varepsilon_0$ shows that $\mu_{CU} = \mu_{PU}$ is satisfied, when $\lambda = \varepsilon_0$. 

Source Sink Stability of $k_2^*$
Fig. 10-1: Poverty traps, when \( \bar{q} < q < q_0 \)

Note:
In Region X, \( k_0^* \) and \( k_2^* \) are saddle points and \( k_1^* \) is a sink.
In Region Y, \( k_2^* \) is a saddle and \( k_1^* \) is a sink.

Fig. 10-2: Poverty traps, when \( \bar{q} < q < q_0 \)
Fig. 11: Situation 2 in $0 < \lambda < \lambda_{NS}$
Note: It should be noted that the economic variables jump from one point to the next along the bold lines shown in this figure, since this dynamical system is discrete-time.
An economy can escape the poverty traps! 

\[ m_i = m_{i+1} \text{ [eq.(5)']} \]

\[ k_1 = k_{i+1} \text{ [eq.(4)]} \]

\[ m_i = m_{i+1} \text{ [eq.(4)']} \]

\[ \lambda_{NS} < \lambda < s - \varepsilon_i \]

and \[ \mu_{NS} < \mu < \mu_{PU} \]
Fig. 14: Situation 2

\[ \lambda_{NS} < \lambda < s - \epsilon_i \]

and \( \mu_{pu} < \mu < \mu_H \)

\[ [\text{eq.}(5)'] \]

\[ [\text{eq.}(4)'] \]