The speed of convergence and alternative government financing

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Abstract

In this paper we are concerned with the speed of convergence in the Sidrauski monetary optimizing model (Sidrauski, Am. Econ. Rev. 57 (1967) 534), in which the steady-state capital stock is not influenced by a change in public spending. The paper studies how different the speed of convergence to the steady state is affected by a change in government spending under the various ways of financing. In exploring the transitional adjustment to the public policy shock, it will be convenient to focus upon the degree of relative risk averse. © 2002 Elsevier Science B.V. All rights reserved.

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Keywords: The speed of convergence; Sidrauski monetary growth model; Alternative government financing; The degree of relative risk averse

1. Introduction

Does the degree of income inequalities across countries tend to shrink over time? It is well understood that the disparities of per-capita income across 47 Japanese prefectures, 48 US states and 90 European regions disappear over time (the phenomenon is called σ-convergence) and on the other hands, the distribution of per-capita income for the set of 110 countries is increasingly unequal (there has been σ-divergence in a world). Do poor economies grow faster than rich ones? Are the countries that are relatively poor now different from the ones that were relatively poor 100 years ago? The questions in which all economists will be interested are closely connected with the concept of β-convergence. It will be convenient that the convergence hypotheses are decomposed into two categories (Galor, 1996). If the above answer is unconditionally yes, then we can find the evidence of absolute β-convergence in the countries.

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The concept discussed next is called conditional $\beta$-convergence. As the distance that separates an economy from its own steady state is larger, the growth rate of per-capita income is higher. The steady-state value of an economy is determined by its own structural characteristics such as preferences, technologies, public policy and population growth rate. A large cross-section of countries actually exhibit the existence of conditional $\beta$-convergence, not but absolute $\beta$-convergence. By contrast, the sample of OECD, the states within the United States, the prefectures of Japan, and regions within several European countries exhibit the strong evidence of absolute $\beta$-convergence. Regardless of time period, the speeds at which the countries or the regions of different countries converge are roughly same: about 2 percent per year. As for the empirical findings above, see Mankiw et al. (1992), Barro and Sala-I-Martin (1992a, b) and Sala-I-Martin (1996a, b). We consider the theoretical interaction between the way that public consumption is financed and the speed of $\beta$-convergence that is regarded as a necessary condition for the existence of $\sigma$-convergence. As mentioned in Backus et al. (1992), the invariance of government policy to long run growth is supported by empirical evidence for OECD economies. Barro and Sala-I-Martin (1995a), Barro et al. (1995b), Ortigueira and Santos (1997) and Turnovsky (2001) conclude that the computed rates of convergence in the non-monetary growth models tend to be larger than the empirical estimated rate, i.e., about 2 percent. If the computed rate of convergence in the Sidrauski monetary growth model is adequately slow, we can suggest that government consumption may nevertheless matter for growth rate over long periods of time.

The idea that different government financing modes may have different impacts on the aggregated economy has a long history in the social science. For example, Turnovsky (1978) postulates the demand functions of money and consumption in conventional ways and compares the comparative static effects of money-finance and bond-finance on the system in a growing economy. Developing the Sidrauski’s monetary growth model with non-interconnected overlapping generations, Alogoskoufis and Ploeg (1994) examine the effects on real growth rate of lump-sum-tax-financed, money-financed and bond-financed increases in government spending. Palivos and Yip (1995), on the other hands, utilize the Stockman’s cash-in-advance model (1981) in the framework of endogenous growth to assess the relative merits of two different modes of financing; tax-financing and money-financing. However, Turnovsky focus upon the long-run effects of alternative government financing modes in a growing economy, while the property of growth models in Ploeg and Alogoskoufis and Palivos and Yip is that there is no transitional dynamics and the economy jumps immediately to the balanced growth path. To put it another way, the analyses of the authors are limited to the long-run effects of different government financing ways, ignoring the speed of convergence to a stationary solution. In thinking of the policy effect on real economy, the speed of convergence yields an important information on relative emphasis that should be placed on the steady-state effect and transitional dynamics one. If the speed of convergence is low, the steady-state effect does not play an adequate role in investigating policy effect as a predictive power. The theoretical interaction between monetary policy and the rate of convergence has been already studied by Fischer (1979), Asako (1983), Cohen (1985) and Abel (1985). It is, however, left as a question whether and how
fiscal policy such as the government financial option affects the speed at which the economy approaches the steady state.

This paper focuses on the transition to the steady state rather than on the property of the steady state. In particular, the steady-state capital stock is determined by the modified golden rule. The value of steady-state capital stock is such that the interest rate is equal to the pure rate of time preference plus the depreciation rate of capital stock and thus is invariant to the levels of government spending. In this model, the degree of relative risk averse plays the central role in ascertaining whether the convergence speed is differently influenced by increasing government spending under the three alternative financing schemes, lump-sum-tax-financing, money-financing and bond-financing. The calibration exercise supplements how different in quantity the comparative dynamic effects are.

This paper is organized as follows. Section 2 describes the individual optimizing behaviors. In Section 3, we derive the basic equations that characterize the dynamic behavior of the economy. Section 4 shows that there exists a unique equilibrium path that converges to the steady state. Section 5 verifies that a larger scale of public spending produces less rapid capital accumulation, regardless of the financing schemes. Section 6 compares the decelerated effect on the rate of convergence under the different schemes of government financing. Section 7 concludes with a summary of results.

2. The structure of model

We normalize the consumers at time 0 to unity and expect the size of consumers to grow at the rate \( n \). Their objective is to select their portfolio of assets as well as their rate of consumption to maximize the value of life time utility as function of per-capita consumption \( c_t \) and per-capita real money balance \( m_t \),

\[
\int_0^\infty \frac{(c_t^\gamma m_t^\delta)}{1 - R} e^{-(\rho - n)t} \, dt, \quad \gamma + \delta \leq 1,
\]

subject to

\[
\dot{k}_t + \dot{m}_t + \dot{b}_t = (r_t - \eta - n)k_t + (r_t^b - \pi_t - n)b_t - (\pi_t + n)m_t + w_t - \tau_t - c_t,
\]

where \( c_t \) is the real private consumption, \( m_t \) the demand for real money balances, \( k_t \) the demand for physical capital, \( b_t \) the demand for real government bonds, \( \tau_t \) the lump-sum taxation, \( w_t \) the real wage rate, \( r_t \) the real rental rate of physical capital, \( \eta \) the capital depreciation rate, \( \pi_t^b \) the nominal rate of interest on government bonds, \( \pi_t \) the anticipated inflation rate, \( \rho \) the consumer’s rate of time preference and \( n \) the population growth rate.

The utility in Eq. (1) must be bounded, when \( c_t \) and \( m_t \) are constant over time. We must assume A.1;

A.1. \( \rho - n > 0 \).
To solve the dynamic optimization, we must set up the Hamiltonian function. The necessary conditions can be derived as

\[ \delta m_t^{(1-R)} - c_t^{(1-R)} e^{-\gamma (\rho-n)t} = \nu_t (\pi_t + r_t - \eta), \]

\[ \gamma m_t^{(1-R)} e^{-\gamma (\rho-n)t} = \nu_t, \]

\[ \dot{\nu}_t = -(r_t - \eta - n), \]

\[ m_t = \left( \frac{\delta}{\gamma} \right) \left( \frac{c_t}{r_t + \pi_t - \eta} \right), \]

\[ r_t - \eta = \nu_t b_t - \pi_t, \]

where \( \nu \) is the costate variable attached to the budget constraint (2).

Firms, on the other hands, combine the services of capital \( K_t \) and current labor \( N_t \) to produce output \( Y_t \), according to a constant return technology given by the production function \( Y_t = F(K_t, N_t) \). Each input exhibits positive and diminishing marginal product. It is convenient to deal with quantities per unit of labor \( y_t \equiv Y_t/N_t \) and \( k_t \equiv K_t/N_t \). The production function can then be written in the intensive form \( y_t = f(k_t) \), where \( f' > 0, \ f'' < 0 \) and \( f(0) = 0 \). We assume that this function satisfies the Inada conditions that \( f' \rightarrow \infty \) as \( k_t \rightarrow 0 \) and \( f' \rightarrow 0 \) as \( k_t \rightarrow \infty \). Under perfect competition, firms equate the marginal productivities of capital and of labor to the real rental rate of capital, \( r_t \) and to the real wage, \( w_t \), respectively. Thus, we can show that

\[ f'(k_t) = r_t \quad \text{and} \quad f(k_t) - f'(k_t)k_t = w_t. \]

Government expenditure must be financed by lump-sum tax and/or printing money and/or issuing bonds. As a policy of government debt, it is assumed that bonds and money are maintained in a fixed ratio

\[ b_t/m_t = \xi. \]

This specification of debt policy is adopted in much of monetary growth literature. See Foley and Sidrauski (1971). Then, the government budget constraint is

\[ \tau_t + \mu(1 + \xi)m_t = g_t + r_t b_t, \]

where \( \mu \) is the rate of monetary growth.

3. The dynamic system

We think of an economy in which there exist two alternative financing schemes, lump-sum-tax financing and mixed money-bond financing, to raise the necessary
government revenue. We derive the dynamic equations of state variables that correspond to the two different financing schemes.

3.1. Tax-financing

Suppose that lump-sum-tax is the residual mode of public finance. Let us consider a set of dynamic equations in this case.

Using Eqs. (4) and (5), the behavior of $c_t$ is

$$\dot{c}_t = \frac{c_t}{1 - (1 - R)\gamma} (r_t - \eta - \rho)$$

$$+ \frac{\delta(1 - R)c_t}{1 - (1 - R)\gamma} \left[ \mu + r_t - \eta - n - \left( \frac{\delta}{\gamma} \right) \left( \frac{c_t}{m_t} \right) \right].$$

From $m_t = M_t/P_tN_t$ and Eq. (6), the motion of $m_t$ can be expressed as

$$\dot{m}_t = \left[ \mu + r_t - \eta - n - \left( \frac{\delta}{\gamma} \right) \left( \frac{c_t}{m_t} \right) \right] m_t.$$ (11)

Combining Eqs. (2), (7) and (8), the equilibrium condition for final goods is described as

$$\dot{k}_t = y_t - c_t - g - (n + \eta)k_t.$$ (12)

Noting that $f'(k_t) = r_t$ and $y_t = f(k_t)$, Eqs. (10)–(12) give a set of dynamic equations with respect to $c_t$, $m_t$ and $k_t$. Hence, these equations constitute a complete dynamic system in the model where lump-sum-tax is required to finance government spending.

In the steady state that $c_t$, $k_t$ and $m_t$ are constant, we can obtain the relationships

$$f'(k^*) = \rho + \eta,$$ (13)

$$f(k^*) = c^* + g + (n + \eta)k^*,$$ (14)

$$\mu + \rho - n = \left( \frac{\delta}{\gamma} \right) \left( \frac{c^*}{m^*} \right).$$ (15)

From Eqs. (13) and (14), we can get the super-neutrality result, because a rise in public spending leads to 100 percent crowding-out of consumption,

$$\frac{dc^*}{dg} = -1.$$ (16)

In other words, $k^*$ is independent of the level of public spending.

3.2. Mixed money-bond financing

Suppose that both money and government bonds are the residual mode of public finance. Let us consider a set of dynamic equations in this case.
Using Eqs. (4), (5) and (18), the behavior of $c_t$ is

$$
\dot{c}_t = \frac{c_t}{1 - (1 - R)\gamma}(r_t - \eta - \rho) + \frac{\delta(1 - R)c_t}{1 - (1 - R)\gamma} \left[ \frac{(g - \tau_t)}{(1 + \xi)m_t} + r_t - \eta - n - \frac{(\delta/\gamma)c_t}{(1 + \xi)m_t} \right].
$$

(17)

From $m_t = M_t/N_t$ and Eqs. (6), (7) and (9), the motion of $m_t$ can be expressed as

$$
\dot{m}_t = \frac{(g - \tau_t)}{(1 + \xi)m_t} + r_t - \eta - n - \frac{(\delta/\gamma)c_t}{(1 + \xi)m_t} m_t.
$$

(18)

The equilibrium condition for AMMnal goods is

$$
\dot{k}_t = y_t - c_t - g - (n + \eta)k_t.
$$

(19)

From $f'(k_t) = r_t$ and $y_t = f(k_t)$, Eqs. (17)–(19) give a set of dynamic equations with respect to $c_t$, $m_t$ and $k_t$. Hence, these equations constitute a complete dynamic system in the model that the mixture of money and bonds is required to finance government spending.

In the steady state, we can easily obtain the relationships

$$
f'(k^*) = \rho + \eta,
$$

(20)

$$
f(k^*) = c^* + g + (n + \eta)k^*.
$$

(21)

$$
\frac{g - \tau}{m^*} + (1 + \xi)(\rho - n) = \left( \frac{\delta}{\gamma} \right) \left( \frac{c^*}{m^*} \right).
$$

(22)

Using Eqs. (20)–(22), we can express the comparative static effects of an increase in $g$ on $c^*$ and $m^*$ as

$$
\frac{dc^*}{dg} = -1,
$$

(23)

$$
\frac{dm^*}{dg} = -\frac{(\gamma + \delta)}{(\rho - n)\gamma(1 + \xi)}.
$$

(24)

In this case, we can also conclude that $k^*$ is independent of the level of $g$, because 100 percent crowding-out of $c^*$ is caused by a rise in $g$.

4. The stability of steady-state equilibrium

4.1. Lump-sum-tax finance

Following Fischer (1979), we assume the following in the case of financing through lump-sum-taxation.
A.2. $\mu + \rho - n > 0$.

To investigate the stability of the steady state, we linearize Eqs. (10)–(12) around $(c^*, m^*, k^*)$. We can express the matrix of the linearized system as

$$
\begin{bmatrix}
-\frac{\delta(1-R)(\mu + \rho - n)}{1 - \gamma(1-R)} & \frac{\gamma(1-R)(\mu + \rho - n)^2}{1 - \gamma(1-R)} & f''c^* \frac{1 + \delta(1-R)}{1 - \gamma(1-R)} \\
-\frac{\delta}{\gamma} & \mu + \rho - n & f''(\frac{\delta}{\gamma})c^* \\
-1 & 0 & \rho - n
\end{bmatrix}
$$

The sum of the characteristic roots of this system is given by the trace of (25),

$$
\text{Trace} = \frac{1 - (\gamma + \delta)(1-R)}{1 - \gamma(1-R)}(\mu + \rho - n) + (\rho - n) > 0.
$$

The product of the roots of the system (25) is described as the determinant,

$$
\text{Det} = \frac{\mu + \rho - n}{1 - \gamma(1-R)} f''(k^*)c^* < 0.
$$

Thus, the system (25) has one negative and two positive roots (possibly complex with real parts positive) and we find that there exists a unique perfect foresight path that converges to the steady state, $(c^*, m^*, k^*)$.

4.2. Mixed money-bond finance

In the case of mixed money-bond financing, we assume the following:

A.3. $R \geq 1$.\(^1\)

To analyze the stability of the steady state, we linearize Eqs. (17)–(19) around $(c^*, m^*, k^*)$, so the matrix of this linearized system can be obtained as

$$
\begin{bmatrix}
-\frac{\delta(1-R)(\delta/\gamma)(c^*/m^*)}{(1 + \xi)[1 - \gamma(1-R)]} & \frac{\delta(1-R)(c^*/m^*)(\rho - n)}{1 - \gamma(1-R)} & f''(k^*)c^* \frac{1 + \delta(1-R)}{1 - \gamma(1-R)} \\
-\left(\frac{\delta}{\gamma}\right) \left(\frac{1}{1 + \xi}\right) & \rho - n & f''(k^* )m^* \\
-1 & 0 & \rho - n
\end{bmatrix}
$$

\(^1\)As $R^{-1}$ is the elasticity of substitution of the composite good of consumption goods and money, the empirical estimates of $R^{-1}$ are within the range (0, 1). See Hall (1988), Epstein and Zin (1991), Patterson and Pesaran (1992) and Ogaki and Reinhart (1998).
The sum of the characteristic roots of this system is given by the trace of (28),
\[
Trace = -\delta(1 - R)(\delta/\gamma)(c^*/m^*)/(1 + \xi)[1 - \gamma(1 - R)] + 2(\rho - n) > 0.
\] (29)

The product of the roots of the system (28) is described as the determinant,
\[
Det = \frac{\rho - n}{1 - \gamma(1 - R)} f''(k^*)c^* < 0.
\] (30)

Thus, the system (28) has one negative and two positive roots (possibly complex with real parts positive) and we find that there exists a unique perfect foresight path that converges to the steady state, \((c^*, m^*, k^*)\).

5. The speed of convergence

Let us examine the effect of a change in government consumption \(g\) on a unique stable root, i.e., the rate of investment.

5.1. Finance through lump-sum-taxation

We investigate how a tax-financed increase in \(g\) has an impact upon a unique negative root in the linearized system (25). We can write the characteristic equation for the linearized system as
\[
\Psi_1(\lambda, g) = -\lambda^3 + \lambda^2 \left[ \frac{1 - (\gamma + \delta)(1 - R)}{1 - \gamma(1 - R)}(\mu + \rho - n) + (\rho - n) \right] - \lambda \left[ \frac{1 - (\gamma + \delta)(1 - R)}{1 - \gamma(1 - R)}(\rho - n)(\mu + \rho - n) \right]
+ f''(k^*)c^* \frac{1 + \delta(1 - R)}{1 - \gamma(1 - R)} + f''(k^*)c^* \frac{\mu + \rho - n}{1 - \gamma(1 - R)} = 0.
\] (31)

Using Eq. (31), we can obtain
\[
\frac{d\lambda}{dg} = -\left( \frac{\partial \Psi_1/\partial g}{\partial \Psi_1/\partial \lambda} \right).
\] (32)

where the derivative is, of course, evaluated at the negative root \(\lambda\). Noting \(\lambda < 0\), we find that \(\partial \Psi_1/\partial \lambda < 0\) and thus
\[
sgn\left( \frac{d\lambda}{dg} \right) = sgn\left( \frac{\partial \Psi_1}{\partial g} \right).
\] (33)

Using Eq. (16), differentiating Eq. (31) with respect to \(g\) yields
\[
\frac{\partial \Psi_1}{\partial g} = \frac{1 + \delta(1 - R)}{1 - \gamma(1 - R)} f''(k^*)\lambda - \frac{\mu + \rho - n}{1 - \gamma(1 - R)} f''(k^*).
\] (34)
Combining Eqs. (31) and (34) and noting $\lambda = 0$, we can obtain the following

\[
\begin{align*}
(\frac{\partial \Psi_1}{\partial g})^c = -\lambda^3 + \lambda^2 \left[ 1 - (\gamma + \delta)(1 - R) \right] \frac{1}{1 - \gamma(1 - R)} (\mu + \rho - n) + (\rho - n) \\
- \lambda \left[ 1 - (\gamma + \delta)(1 - R) \right] (\rho - n)(\mu + \rho - n) > 0.
\end{align*}
\]

(35)

Considering Eqs. (33) and (35), we can conclude

\[
\frac{d\lambda}{dg} > 0.
\]

(36)

This outcome can be also summarized as follows.

**Proposition 1.** By contrast with the steady state, a tax-financed increase in public spending reduces the rate of investment for any given level of the capital stock.

In the case of unitary risk aversion, i.e., $R = 1$, characteristic equation (31) is that

\[
\Psi_1(\mu, \lambda) = -[\lambda - (\mu + \rho - n)] \cdot \psi_1 = 0,
\]

where

\[
\psi_1 \equiv [\lambda^2 - (\rho - n)\lambda + f''(k^*)c^*].
\]

(37)

Solving the equation above, $\psi_1 = 0$, we can obtain the stable root, which is not independent of the level of $g$.

5.2. Mixed money-bond finance

In the case where government finances the deficits by printing money as well as by issuing bonds, we examine the effect of an increase in $g$ on the unique negative root obtained in the system (28). The characteristic equation for the system (28) can be described as

\[
\Psi_2(\lambda, g) = -\lambda^3 + \lambda^2 \left[ -\delta(1 - R) \right] \frac{1}{1 - \gamma(1 - R)} \left( \frac{c^*}{m^*} \right) \left( \frac{1}{1 + \xi} \right) + 2(\rho - n) \\
- \lambda \left[ -\delta(1 - R) \right] \frac{1}{1 - \gamma(1 - R)} \left( \frac{c^*}{m^*} \right) \left( \frac{1}{1 + \xi} \right) (\rho - n) \\
+ c^* f''(k^*) \frac{1 + \delta(1 - R)}{1 - \gamma(1 - R)} + (\rho - n)^2 \\
+ \frac{\rho - n}{1 - \gamma(1 - R)} c^* f''(k^*) = 0.
\]

(38)

Using Eqs. (16) and (24), differentiating Eq. (38) with respect to $g$ yields

\[
\begin{align*}
(\frac{\partial \Psi_2}{\partial g})^c = -\frac{\delta(1 - R)}{1 - \gamma(1 - R)} \left( \frac{\delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right)
\end{align*}
\]
\[
\times \left[ -\left( \frac{c^*}{m^*} \right) + \left( \frac{c^*}{m^*} \right)^2 \left( \frac{\gamma + \delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right) \left( \frac{1}{\rho - n} \right) \right] \lambda^2 \\
- \left[ \frac{\delta(1 - R)}{1 - \gamma(1 - R)} \left( \frac{\delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right) \right] \\
\times \left\{ \left( \frac{c^*}{m^*} \right) (\rho - n) - \left( \frac{c^*}{m^*} \right)^2 \left( \frac{\gamma + \delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right) \right\} \\
- \frac{1 + \delta(1 - R)}{1 - \gamma(1 - R)} \left[ \frac{f''(k^*)c^*}{\lambda} - \frac{\rho - n}{1 - \gamma(1 - R)} f''(k^*)c^* \right].
\] (39)

Noting A.2 and \( \lambda < 0 \), combining Eqs. (38) and (39) results in
\[
(\partial \Psi_2/\partial g)c^* = -\lambda^3 + \Phi_1 \lambda^2 + \Phi_2 \lambda > 0,
\]
where
\[
\Phi_1 = -\frac{\delta(1 - R)}{1 - \gamma(1 - R)} \left( \frac{\delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right) \left( \frac{c^*}{m^*} \right)^2 \left( \frac{\gamma + \delta}{\gamma} \right) \left( \frac{1}{\rho - n} \right) + 2(\rho - n) > 0,
\]
\[
\Phi_2 = \left[ \frac{\delta(1 - R)}{1 - \gamma(1 - R)} \left( \frac{\delta}{\gamma} \right) \left( \frac{1}{1 + \xi} \right) \left( \frac{c^*}{m^*} \right)^2 \left( \frac{\gamma + \delta}{\gamma} \right) - (\rho - n)^2 \right] < 0.
\] (40)

Hence, we can conclude \( d\lambda/dg > 0 \).

**Proposition 2.** The rate of investment is less rapid, as public consumption, which is financed by the mixture of money and bonds, is larger.

When \( R = 1 \), Eq. (38) reduces to
\[
\Psi_2(\lambda, g) = -[\lambda - (\rho - n)]\psi_2 = 0,
\]
where
\[
\psi_2 = \psi_1 \equiv \left[ \lambda^2 - (\rho - n)\lambda + f''(k^*)c^* \right].
\] (41)

Thus, we can summarize the case of \( R = 1 \) as follows:

**Proposition 3.** In the unitary risk averse, the decelerated effect on the rate of capital accumulation does not make any difference in quantity among three different financing regimes, bond-financing, money-financing and tax-financing.

6. Comparisons

We calibrate the models in the previous section and use the calibrated models to estimate how different the alternative government financing schemes affect a unique
Table 1
Parameter values used in the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(k_t)$</td>
<td>$k_0^{0.3}$</td>
<td>Production technology</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.05</td>
<td>Annual rate of time preference</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.01</td>
<td>Population growth rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.05</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$R$</td>
<td>2</td>
<td>Degree of relative risk averse</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>Weight of consumption in utility</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.3</td>
<td>Weight of money in utility</td>
</tr>
<tr>
<td>$g$</td>
<td>0.52524</td>
<td>Government spending</td>
</tr>
</tbody>
</table>

Table 2
Tax-financing

<table>
<thead>
<tr>
<th>Sensitivity of $d - \lambda_1/dg$ to changing $\mu$</th>
<th>Stable root $\lambda_1$</th>
<th>$d - \lambda_1/dg$</th>
</tr>
</thead>
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<tr>
<td>$\mu$</td>
<td>$\lambda_1$</td>
<td>$d - \lambda_1/dg$</td>
</tr>
<tr>
<td>0.01</td>
<td>$5.9739 \times 10^{-2}$</td>
<td>$9.7669 \times 10^{-2}$</td>
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<td>0.02</td>
<td>$7.2045 \times 10^{-2}$</td>
<td>$9.7149 \times 10^{-2}$</td>
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<td>0.03</td>
<td>$8.5529 \times 10^{-2}$</td>
<td>$9.5443 \times 10^{-2}$</td>
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<td>0.04</td>
<td>$9.6378 \times 10^{-2}$</td>
<td>$6.2003 \times 10^{-3}$</td>
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<tr>
<td>0.05</td>
<td>$0.10226 \pm 5.7176 \times 10^{-1}$</td>
<td>$-5.8649 \times 10^{-2}$</td>
</tr>
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<td>0.06</td>
<td>$0.10159, 0.11472$</td>
<td>$-5.8659 \times 10^{-2}$</td>
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<tr>
<td>0.07</td>
<td>$0.10035, 0.12773$</td>
<td>$-5.8669 \times 10^{-2}$</td>
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<tr>
<td>0.08</td>
<td>$0.13997, 0.9885 \times 10^{-2}$</td>
<td>$-5.8677 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.09</td>
<td>$0.15198, 0.9647 \times 10^{-2}$</td>
<td>$-5.8684 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.10</td>
<td>$0.16391, 0.9488 \times 10^{-2}$</td>
<td>$-5.869 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

stable root $\lambda_1$ and thus the rate of capital accumulation. Table 1 lists the values of parameters that we have used in the simulations. Most of these are relatively standard and are similar to the values used in other calibrated models.

First, we consider the case in which public spending is financed by accommodating lump-sum taxation. Table 2 reports the results of a sensitivity analysis in which we vary the rate of monetary growth, the parameter $\mu$, from 0.01 up to 0.1; all other parameters are set at the values described in Table 1. The table tabulates the eigenvalues of the matrix and estimates the negative effect of an increase in $g$ on $-\lambda_1$. The extent to which increasing $g$ influences $-\lambda_1$ is extremely small. The negative effects are slightly sensitive to the choice of $\mu$. As $\mu$ is increased, the negative effect of an increase in $g$ on $-\lambda_1$ is larger. We should note that $-\lambda_1$ or the rate of investment increases with $\mu$. See Fischer (1979). Thus, the effectiveness of fiscal policy varies proportionately with the rate of real growth.

Next, suppose the case in which public spending is financed by printing money and by issuing bonds. When we vary the parameter $\zeta$, all other parameters are set

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2 Table 2 confirms the existence of Fischer’s proposition (1979), but the degree to which the expansion of monetary growth contribute the convergence rate is negligibly small.
Table 3
Mixed money-bond financing

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>Roots 1 and 2</th>
<th>Stable root ( \lambda )</th>
<th>( d - \lambda_1/dg )</th>
</tr>
</thead>
<tbody>
<tr>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1.25</td>
<td>( 7.3379 \times 10^{-2} \pm 2.307 \times 10^{-2}i )</td>
<td>(-5.9700 \times 10^{-2})</td>
<td>(5.1034 \times 10^{-2})</td>
</tr>
<tr>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: We set the value of \( \tau = 0.52524 \).

at the values described in Table 1; Table 3 reports the values of the roots of matrix and estimates the negative effect of an increase in \( g \) on \(-\lambda_1\). We think of the mixed money-bond finance where government finances the deficit, using a larger ratio of bonds to money and investigate how the transitional effects are influenced by this government decision. Noting Eq. (22), we can see that \( (1 + \xi)m^* \) is invariant to the level of \( \xi \). Eq. (38) verifies that the value of a stable root is independent of \( \xi \). Moreover, Eqs. (38) and (40) show that the negative effect on \(-\lambda_1\) of an increase in \( g \) is also unaffected by a change in \( \xi \). The simulations in Table 3 confirm that within any range of \( \xi \), an increase in \( g \) by a factor of 1 reduces the rate of capital accumulation by a factor of \( 5.1034 \times 10^{-2} \). The extent to which the rate is affected by increasing \( g \) under mixed money-bond financing is also small. Let us summarize the relationship between bond- and money-financing.

**Proposition 4.** Whether public spending is financed by issuing bonds or by printing money does not affect the amount of decelerated effect on the growth rate of capital stock, even when \( R \neq 1 \).  

We compare the quantitative dynamic effects of a rise in \( g \) under three alternative regimes of financing; an increase in the lump-sum tax, an increase in inflation tax and an increase in government bonds.

**Proposition 5.** For the wide range of money growth rate, an increase in seigniorage or in government bonds leads to a larger fall in the growth rate of capital stock than an increase in lump-sum-taxation.

If we set \( \delta = 0 \) and \( \gamma = 1 \) in this monetary growth model, the model simplifies to be Ramsey non-monetary growth model. Then, the rate of convergence is \( 5.836 \times 10^{-2} \).
As is well-known, the rates of convergence that are calculated in the Ramsey model tend to be much larger than ones that are empirically estimated. Tables 2 and 3 show that the rates are not reduced to the empirical plausible values, but are slightly raised, even if we consider the monetary sector in the Ramsey model by introducing money into the utility function.

7. Concluding remarks

Developing the Sidrauski intertemporal optimizing model in which the steady-state capital stock is invariant to the level of public spending, we analyze how the speed of convergence to the steady state is affected by an increase in government consumption under the three financing methods, lump-sum tax, inflation tax and government bonds. Regardless of the financing schemes, the convergence rate of capital stock is reduced by an increase in government spending. The degree of relative risk averse is particularly significant, when we assess the quantitative differences caused by the option of government financing methods. We can report the results of the analysis as follows: In the unitary risk averse, i.e., when \( R = 1 \), these three financing modes do not cause any quantitative difference in the decelerated effect on the rate of convergence. Even if \( R \neq 1 \), it makes no difference in quantity between bond- and money-financing.

The calibration reports that a rise in public spending by a factor of 1 reduces the rate of capital accumulation by a factor of \( 5.1034 \times 10^{-2} \), the amount of which is not large enough to emphasize the effectiveness of fiscal policy. In the case where public spending is financed by accommodating lump-sum taxes, the decelerated effect varies proportionately with the rate of monetary growth \( \mu \); it varies from \( \mu = 0.01 \) to 0.1. For the wide range of values of monetary growth rate, nevertheless, a bond- or a money-financed increase in public spending reduces the growth rate of capital more than a tax-financed increase. As in the non-monetary Ramsey model, the speed of convergence is also high in the calibrated monetary growth model. Thus, we cannot conclude that public spending as well as money growth rate influence the performance of real economy over long periods of time.

In this model, however, there are still some works that should be undertaken. The property of transitional dynamics in our model is that the dynamic adjustment locus is a one-dimensional stable manifold and all variables converge at identical and constant rates. Therefore, we can focus on the capital accumulation and its constant speed of convergence during the transition. However, it is more realistic and general to allow the convergence speeds to vary across variables and time (Dowrick and Nguyen, 1989; Bernard and Jones, 1996). Unlike this paper, Eicher and Turnovsky (1999, 2001) show that the transitional behaviors in a two-sector R& D-based non-scale growth model are characterized by a two-dimensional stable saddle path, the presence of which can generate the convergence property that is supported by the empirical works mentioned just above. We need to study the role of government policy in the growth model that the transitional dynamics are more consistent with the empirical findings.
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References


