Indeterminacy, labor and capital income taxes, and nonlinear tax schedules

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Abstract

Using a finance-constrained model, as in Barinci and Cheron (2001), this paper examines the role of procyclical and countercyclical tax rates on labor and capital income in aggregate fluctuations driven by the beliefs of agents. The analysis shows that the cyclicity of labor income tax rate has the monotonically negative impact on the possibility of indeterminacy, while the non-monotonic relations exist between the cyclicity of capital income tax rate and the likelihood of indeterminacy. It is shown that labor and capital income taxes have remarkably different impacts on the probability of indeterminacy for a sufficiently wide range of variability.

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1 Introduction

It is important to examine how the variability of income tax rate has an impact on the probability of indeterminacy in dynamic general equilibrium models when evaluating the effectiveness of government tax policy as an "automatic stabilizer". Indeterminacy here means that the equilibrium paths converging to the same steady state are innumerable, and so we can endogenously explain aggregate fluctuations with self-fulfilling changes in agents' beliefs about the future. More specifically, we need not rely on persistently exogenous shocks relating to economic fundamentals to explain the occurrence of economic fluctuations.

Indeterminacy itself is possible in standard optimal growth models by considering the increasing returns arising from, for example, external effects in production. For instance, in a one-[two]-sector growth model, Benhabib and Farmer (1994) [(1996)] pointed out the importance of production externalities for the emergence of indeterminacy and showed that indeterminacy emerges with a sufficiently strong [relatively mild] degree of increasing returns. In related work, Barinci and Cheron (2001) clarified that endogenous fluctuations arise under weaker increasing returns using Woodford's finance-constrained model (1986). Drawing on the above, many economists have explored how government tax policy affects the range of parameter values associated with indeterminacy to design a stabilizing tax policy against agents' belief-driven aggregate fluctuations.

Several studies have already examined these issues using the one- and two-sector Ramsey models. Schmitt-Grohe and Uribe (1997), for example, pointed out the possibility that indeterminacy arises in the one-sector model without any production externalities if labor income tax rates endogenously adjust to finance the preset level of government expenditure. Schmitt-Grohe and Uribe (1997) found that countercyclical income taxes are important factors affecting the emergence of endogenous fluctuations. Similarly, Guo and Lansing (1998) considered nonlinear tax rates on income, and investigated the relation between the likelihood of indeterminacy and the slope indexing the degree of income tax progressiveness. The conclusion is that the minimum values of increasing returns leading to indeterminacy are higher as income taxes become more progressive. Accordingly, progressive income taxes stabilize the economy by making indeterminacy less likely. Finally, using the two-sector model, Guo and Harrison (2002) drew a conclusion contrary to the one-sector model in their view that progressive income taxes raise the probability of indeterminacy for most reasonable values of production externalities.

A common feature in Guo and Lansing (1998) and Guo and Harrison (2002) is that there is no attempt to distinguish between labor and capital income taxes. Although the current paper does not employ the Ramsey model, rather Woodford’s finance-constrained model (1986) with production externalities, as in Barinci and Cheron (2001), we consider nonlinear income taxes in which labor and capital income tax rates are separated, and compare how the variability of income tax rates influences the appearance of agents’ belief-driven aggregate fluctuations.

\footnote{Based on the one-sector Ramsey model, Guo (1999) clearly separates labor and capital income. However, Guo (1999) emphasizes only the importance of progressive labor income taxes to stabilize the economy and not how progressive labor and capital income taxes variously influence the likelihood of indeterminacy, as here.}
belief-driven aggregate fluctuations. The present paper clarifies the appropriateness of such a distinction between labor and capital income taxes, because labor and capital income taxes have strikingly different effects on the likelihood of indeterminacy across a relatively wide range of variability.

Using a finance-constrained model (1986) as in Barinci and Cherón (2001), Gokan (2008) argued that indeterminacy is more (less) likely if labor (capital) income taxes are endogenously determined to finance the preset level of government expenditure. However, while Gokan (2008) distinguished between tax rates on labor and capital income, he considered endogenous income taxes associated only with countercyclical income taxes. Therefore, the present paper considers a wider range of cyclical income tax rates.

Further, some studies have also considered how the variability of tax rates influences the appearance of indeterminacy in a finance-constrained model, as studied in Grandmont et al. (1998). For example, Dromel and Pintus (2006) explored how progressive tax rates on labor income influence the range of elasticity of capital–labor substitution that induces indeterminacy. However, Dromel and Pintus (2006) focused almost exclusively on constant returns to scale in production and completely ignored progressive tax rates on capital income. Thus, Dromel and Pintus (2006) did not reach the conclusion in this paper that tax rates on labor and capital have remarkably different impacts on the likelihood of indeterminacy. Lloyd-Braga et al. (2008) likewise considered technology only with constant returns to scale and also ignored cyclical tax rates on capital income. Unlike Dromel and Pintus (2006) and Lloyd-Braga et al. (2008), we compare how the variability of labor and capital income tax rates affects the emergence of indeterminacy.

The present paper focuses mainly on cyclical tax rates on labor and capital income that positively or negatively change with their own tax base in a finance-constrained model with production externalities, as in Barinci and Cherón (2001). We explore how cyclical income taxes affect the minimum values of increasing returns leading to indeterminacy. The findings show that the cyclicality of labor income tax rate has the monotonically negative impact on the possibility of indeterminacy, but the non-monotonic relations exist between the cyclicality of capital income tax rate and the likelihood of indeterminacy. The strikingly different results arise with labor and capital taxes when evaluating the stabilizing effects of cyclical income tax rates. Therefore, distinguishing between labor and capital income taxes is meaningful when examining how the cyclicality of income tax rates affects the occurrence of endogenous fluctuations because of the changes in agents’ self-fulfilling expectations.

Moreover, we also examine how progressive and regressive tax rates on labor and capital influence the emergence of indeterminacy, and the obtained results are quite the same as in cyclical income taxes. To my best knowledge, the current paper is the first to do the comprehensive comparisons how the variability of labor and capital income tax rates influences the appearance of indeterminacy.

The rest of the paper is organized as follows. Section 2 analyzes the structure of the

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2Dromel and Pintus (2006) considered progressive and regressive tax rates on income where agents’ choices affect the marginal tax rates, while Lloyd-Braga et al. (2008) considered procyclical and countercyclical tax rates where, while income still affects the marginal income tax rates, agents view the marginal tax rates as independent of their own actions.
model. Section 3 derives the dynamic equations in the market equilibrium. Sections 4
and 5 investigate whether more procyclical tax rates on labor and capital income increase
or decrease the minimum sizes of increasing returns generating indeterminacy. Section
6 describes the policy implications of cyclical income tax rates and Section 7 provides
the intuition underlying the results. Section 8 compares how progressive and regressive
income taxes influence local dynamics near a steady state. Section 9 presents some brief
concluding remarks.

2 Framework

2.1 Workers

We focus directly on an overlapping generations structure with an infinite lifetime arising
from the representative agent model as in Barinci and Cherón (2001). It is well under-
stood that the corresponding equations reflect the actual dynamics of the representative
agent model near monetary steady states only if workers discount the future more than
capitalists. See Appendix A.

In this case, identical workers behave like two-period living agents and participate
in the market in two periods. In the early period, they supply a variable quantity of
labor hours, save their after-tax income by holding outside money, and choose the fol-
lowing period’s consumption. Workers maximize their utility at date \( t \),

\[
ct^{-\sigma} / (1 - \sigma) - l_t^{1+\xi} / (1 + \xi),
\]

subject to the current and anticipated budget constraint,

\[
p_{t+1}c_{t+1} = (1 - \tau_{wt}) p_t w_t l_t = M_{t+1}.
\]

Here, \( c_{t+1} \) is the next period’s consumption, \( l_t \) is the labor supply, \( p_i \) is the \( i \)th period
price of consumption, \( M_{t+1} \) is the nominal outside money held at the end of this period,
\( w_t \) is the real wage rate, and \( \tau_{wt} \) is the variable tax rate on labor income. Following Guo
and Lansing (1998) and Guo (1999), we specify this as follows:

\[
\tau_{wt} = 1 - \eta_w \left( \frac{w_t}{\bar{w} l_t} \right) \phi_w, \quad \eta_w \in (0, 1)
\]

where \( \bar{w} l \) is the base level of labor income taken as given, set here equal to the steady-
state value of labor income. Thus, in the steady state, the tax rate on labor income equals
\( 1 - \eta_w \). It is appropriate to regard that \( 0 < \tau_{wt} < 1 \) is satisfied, since we pay attention to
the economic variables near the stationary state. \( \phi_w \) denotes the slope of the government
tax schedule for labor income. If \( \phi_w > (<)0 \), the labor income tax rate \( \tau_{wt} \) increases
(decreases) with the taxable labor income, and the tax schedule with respect to labor
income is then procyclical or progressive (countercyclical or regressive).

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3If we use a more general utility function, such as \( u (c_{t+1} / B) - v (l_t) \), to satisfy plausible properties,
we can verify the existence of a normalized steady state by appropriately choosing the scaling parameters
\( A \) in production function (3) and \( B \) in the utility function. The consequences in the present paper are
equally true of those in the normalized steady state.
In the case of labor income taxes, the marginal tax rate is:

\[ \tau_{wt}^m = \frac{\partial (\tau_{wt} w_t l_t)}{\partial (w_t l_t)} = 1 - (1 - \phi_w) (1 - \tau_{wt}) . \]

Noting that \( \tau_{wt} \) exists in the range (0,1), \( \phi_w \) should be less than one so as to give the incentive to supply labor given nonlinear labor income taxes. \( \phi_w < 1 \) must be satisfied to eliminate an extreme case of \( \tau_{wt}^m > 1 \). Thus, let us assume the following.

**Assumption 1 \( \phi_w < 1 \).**

Until in Section 7, we focus on the cyclical income taxes that labor income affects the marginal tax rates on labor income, but agents view the marginal tax rates as independent of how much labor they supply to the market. We then obtain the workers’ offer curve as:

\[ c_{t+1} = l_t^\Psi, \text{ where } \Psi \equiv \frac{1 + \xi}{1 - \sigma} . \] (2)

To obtain the gross substitutability of consumption and leisure in (2), let us assume:

**Assumption 2 0 < \sigma < 1 and \( \xi > 0 \Rightarrow \Psi > 1 \)**

Noting that the elasticity of labor supply with respect to the after-tax real wage rate is \( 1/ (\Psi - 1) \), Assumption 2 also ensures that the elasticity of the labor supply is positive.

### 2.2 Production technology

Consumption goods \( y_t \) are produced by combining labor \( l_t \) and the capital stock \( k_t \). The production function exhibits Cobb–Douglas technology as:

\[ y_t = A k_t^{a} l_t^{\beta}, \text{ where } a + b = 1, \ a > a \text{ and } \beta > b, \] (3)

where \( \bar{k}_t \) and \( \bar{l}_t \) denote the average economy-wide use of capital and labor, taken as parametric by individual capitalists, and \( a - a \) and \( \beta - b \) represent the degrees of the external effects derived from the average economy-wide use of capital and labor, respectively. The capital externality can be explained by Arrow’s learning-by-doing, while the thick market hypothesis modeled by Diamond (1982) is sufficient justification for the labor externality.

We focus on the symmetric equilibrium where \( k_t = \bar{k}_t \) and \( l_t = \bar{l}_t \) are satisfied. Defining the real rental rate of capital as \( r_t \), we obtain:

\[ r_t = a A k_t^{\alpha-1} l_t^{\beta} \equiv r (k_t, l_t) \text{ and } w_t = b A k_t^{\alpha-1} l_t^{\beta-1} \equiv w (k_t, l_t) . \] (4)

We assume that the production technology does not generate endogenous growth:

**Assumption 3 \( \alpha < 1 \).**

Assumption 3 can be regarded as an empirically plausible assumption. See Basu and Fernald (1995, 1997).
2.3 Capitalists and government

Here, we consider the behavior of identical capitalists. Unlike workers, they hold no labor and thus do not supply their labor to the market. They would like to hold capital as a storage means of wealth, but choose not to hold outside money as a substitutable asset at a steady state and nearby, because the real return on capital is higher than the real return on money balances.\(^4\)

Until in Section 7, we consider the cyclical income taxes where capital income affects the marginal capital income tax rate, but capitalists view the marginal tax rate as independent of how much they invest. If the utility function of capitalists is logarithmic, Appendix A shows that the optimal behavior of capitalists is:

\[
k_{t+1} = \gamma \left[ (1 - \delta) k_t + (1 - \tau_{rt}) r_t k_t \right],
\]

\[
c^c_t = (1 - \gamma) \left[ (1 - \delta) k_t + (1 - \tau_{rt}) r_t k_t \right],
\]

where \(\gamma\) is the capitalist discount rate, \(\delta\) is the depreciation rate of the capital stock \((0 < \gamma, \delta < 1)\) and \(\tau_{rt}\) is the nonlinear tax rate on capital income specified by:

\[
\tau_{rt} = 1 - \eta_r \left( \frac{r_k}{r_t k_t} \right)^{\phi_r}, \eta_r \in (0, 1)
\]

where \(r_k\) is the base level of capital income set equal to the steady-state value. The steady-state tax rates on capital income are \(1 - \eta_r\). It is appropriate that \(0 < \tau_{rt} < 1\) is satisfied for the same reason as in labor income taxes. \(\phi_r > (<) 0\) implies the procyclical or progressive (countercyclical or regressive) tax schedule with respect to taxable capital income, such that capitalists with income above \(r_k\) face higher (lower) tax rates than those with income below \(r_k\). For the same reason as in Assumption 1, we assume the following:

**Assumption 4** \(\phi_r < 1\).

Finally, the behavior of the government sector requires specification. Noting that \(\tau_{wt}\) and \(\tau_{rt}\) evolve according to (1) and (7) respectively, government expenditure endogenously adjusts to satisfy the government budget:

\[
\frac{M_{t+1} - M_t}{p_t} + (\tau_{rt} r_t k_t + \tau_{wt} w_t l_t) = g_t,
\]

where \(\mu = \frac{M_{t+1} - M_t}{M_t}\) denotes the money growth rate fixed by the government. If \(\mu = 0\), the budget constraint (8) is identical to Guo and Lansing (1998) except that we explicitly separate the labor and capital income taxes. Following the existing works analyzing

\(^4\)In the steady state, the real return on money \(p_t/p_{t+1}\) is \(1/(1 + \mu)\) and the return on capital \(1 - \delta + (1 - \tau_{rt+1}) r_{t+1}\) is \(1/\gamma\). Thus, the former is larger than the latter for \(0 < \gamma < 1\), if the money growth rate is positive or the absolute value of the negative rate of monetary growth is not extremely large.
nonlinear income taxes, we do not consider the cases that government spending enhances the productivity of private capital and raises the utility of agents.

3 Market equilibrium and the dynamic equations

The purpose of the present paper is to examine whether there exist the distinct dynamic implications of the nonlinear tax schedules with respect to labor and capital income. For this purpose, let us derive the dynamic equations of \((k_t, l_t)\) in market equilibrium. Noting that cyclical income taxes are considered, the following can be obtained:

\[
k_{t+1} = \gamma \left[ \eta_r (r k_t)^{\phi_r} (r (k_t, l_t) k_t) ^{1-\phi_r} + (1-\delta) k_t \right], \tag{9}
\]

\[
\eta_w (w l_t)^{\phi_w} (w (k_t+1, l_t+1) l_{t+1}) ^{1-\phi_w} = (1 + \mu) l_t^\gamma. \tag{10}
\]

Equations (9) and (10) determine the equilibrium dynamics of labor and capital for a given initial value of capital. Using (5), (6), (8), (10) and \(y_t = w_t l_t + r_t k_t\), which is the accounting identity describing the complete distribution of total income between labor and capital, we can obtain the equilibrium condition of the goods market, that is \(y_t = (c_t^* + c_t) + k_{t+1} - (1-\delta) k_t + g\).

In the steady-state equilibrium, the economic variables are constant over time—that is, \(l_t = l_{t+1} = l^*\) and \(k_t = k_{t+1} = k^*\) are satisfied (where * denotes the steady-state value). Note that the base levels of labor and capital income are equal to the steady-state values. From (9) and (10), we can then obtain:

\[
\frac{\eta_w}{\eta_r} \left( \frac{b}{a} \right) \left[ \frac{1}{\gamma} - (1-\delta) \right] k^* = (1 + \mu) c^*, \tag{11}
\]

\[
\eta_r a A (k^*)^{\alpha-1} (c^*)^{\beta/\gamma} = \frac{1}{\gamma} - (1-\delta) \tag{12}
\]

where \(c^* = (l^*)^\gamma\). As the cyclical parameters \((\phi_w, \phi_r)\) do not appear in (11) and (12), the arguments regarding the existence of the steady state are equally true of those in Gokan (2008). See Section 7 and Appendix H in Gokan (2008), where it is shown that the steady state \((k^*, l^*)\) is uniquely determined for any sizes of increasing returns in production, \(\alpha\) and \(\beta\).

Next, let us investigate the local dynamic behaviors around the steady state \((k^*, l^*)\). The procedure usually used to study the local stability of the stationary point is the linear map associated with the Jacobian matrix of (9) and (10) evaluated at the fixed point.

\[\text{If the monetary policy rule } \frac{M_{t+1} - M_t}{M_t} = \mu \text{ and the cash-in-advance constraint } \frac{M_t}{p_t} = c_t \text{ are substituted into the workers’ budget constraint } \frac{M_{t+1} - M_t}{p_t} + \frac{M_t}{p_t} = (1 - \tau_{wt}) w_t l_t, \text{ we can obtained } (1 - \tau_{wt}) w_{t+1} l_{t+1} = (1 + \mu) l_t^\gamma. \text{ Substituting (1) in this equation yields (10).}
\]

\[\text{As for the money-market equilibrium, the supply of money is the sum of newly created money } \mu M_t \text{ plus money held by old workers } M_t, \text{ while the demand for money equals } (1 - \tau_{wt}) p_t w_t l_t. \text{ The workers’ budget constraints mean that the money market is in equilibrium.}\]
The linearized dynamics for the deviation $dk = k - k^*$ and $dl = l - l^*$ are determined by:

\[
\begin{bmatrix}
    dk_{t+1} \\
    dl_{t+1}
\end{bmatrix} = \begin{bmatrix}
    \Delta \\
    -\Delta \left( \frac{\alpha}{\beta} \right) \left( \frac{r}{k^*} \right)
\end{bmatrix} \begin{bmatrix}
    \Theta \\
    -\Theta \alpha \left( \frac{r}{r^*} \right) \left( \phi_w - 1 \right) + \Psi \beta \left( \frac{r}{r^*} \right)
\end{bmatrix} \begin{bmatrix}
    dk_t \\
    dl_t
\end{bmatrix},
\]  

(13)

where $\Delta \equiv \gamma (1 - \delta) - (\phi_r - 1) \alpha [1 - \gamma (1 - \delta)]$ and $\Theta \equiv - (\phi_r - 1) \beta [1 - \gamma (1 - \delta)] \left( \frac{r}{r^*} \right)$. The associated Jacobian matrix evaluated at the steady state has trace $T$ and determinant $D$:

\[
T = \gamma (1 - \delta) + \frac{\Psi}{\beta (1 - \phi_w)},
\]

(14)

\[
D = \left[ \gamma (1 - \delta) - (\phi_r - 1) \alpha [1 - \gamma (1 - \delta)] \right] \cdot \frac{\Psi}{\beta (1 - \phi_w)}.
\]

(15)

Needless to say, the sum of the two eigenvalues of the Jacobian matrix is equal to the value of $T$, and the product of the eigenvalues equals the value of $D$. Equations (14) and (15) show that the local dynamics are independent of the level parameters of the income taxes $\eta_w$ and $\eta_r$, but can be significantly affected by the slopes of the tax schedules that index the degree of cyclicality $\phi_w$ and $\phi_r$. These outcomes are consistent with Guo and Lansing (1998) and Gokan (2008).\(^7\) If we set $\phi_w = 0$ and $\phi_r = 0$, the model in this paper reduces to that in Gokan (2008), and thus we can depict the stability property in the plane $(\alpha, \beta/\Psi)$ as in Figure 1. Gokan (2008) showed that because the level parameters $\eta_w$ and $\eta_r$ have no impact on the local dynamics, Figure 1 is also the same as Barinci and Cherom (2001) where government tax policy is not considered.

To examine the dynamic effects of the government tax policy, let us investigate how the region of indeterminacy in Figure 1 is quantitatively affected by changes in the slopes of the government tax schedules $\phi_w$ and $\phi_r$. From (14), (15), Assumptions 1 and 4, the trace $T$ and the determinant $D$ are always positive. For the range of parameter values, it is sufficient to check Lemma 1 to obtain the local stability property.

**Lemma 1** We can verify:

sgn $\left[ D - T - 1 \right] =$ sgn $\left[ \left( \frac{1 - \phi_r}{1 - \phi_w} \right) \alpha + \frac{\beta}{\Psi} - \left( \frac{1}{1 - \phi_w} \right) \right],$

\[
sgn \left[ D - 1 \right] = sgn \left[ [1 - \gamma (1 - \delta)] \left( \frac{1 - \phi_r}{1 - \phi_w} \right) \alpha + \frac{\gamma (1 - \delta) - \beta}{\Psi} \right].
\]

**Proof.** We can obtain Lemma 1 using (14) and (15). \(\blacksquare\)

Sections 4 and 5 analyze how the range of indeterminacy is affected by changing the degree of cyclicality of the labor and capital income tax rates. From Figure 1, labor

\(^7\) As pointed out in Guo and Lansing (1998), this property also holds in the Ramsey’s optimal growth model with increasing returns. Using a finance-constrained model with increasing returns, Gokan (2008) showed that the levels of constant income tax rates have no impact on local behavior, though he considers only countercyclical tax schedules.
externalities needed for the emergence of indeterminacy are relatively high for capital externalities greater than \( \Phi \equiv \frac{1-\gamma(1-\delta)}{2-\gamma(1-\delta)} \). We can identify that \( \alpha > \Phi \) is satisfied for empirically plausible values of the parameters.\(^8\) We explore whether the existence of the procyclical and countercyclical income taxes increases or decreases the minimum values of labor externalities for a given realistic value of capital externalities. We label this value of capital externalities as \( \alpha_0 \) and can assume:

**Assumption 5** \( \alpha_0 > \Phi \).

To facilitate comparison, Section 4-1 considers the nonlinear tax rates only applied to labor income: \( (\phi_w < 1 \text{ and } \phi_r = 0) \), while Section 4-2 focuses on the tax rate variability only for capital income: \( (\phi_w = 0 \text{ and } \phi_r < 1) \).

## 4 Tax rate variability for labor and capital income

### 4.1 Cyclical labor income taxes \( \phi_w < 1 \) and \( \phi_r = 0 \)

Let us introduce only cyclical features into the labor income tax rate. If the labor income tax rate is countercyclical, i.e., \( \phi_w < 0 \), the tax rate \( \tau_w \) decreases, when the tax base \( w_t l_t \) expands. The tax rate increases with the tax base if the tax rate is procyclical, i.e., \( 0 < \phi_w < 1 \). In contrast, the tax rate on capital income \( \tau_r \) is constant, i.e., \( \phi_r = 0 \).

Figure 2 illustrates the qualitative consequences and shows an expansion in the region of externalities generating indeterminacy when \( \phi_w \) is increased in the range \((-1, 1)\). For any given value of capital externalities, the minimum value of labor externalities leading to indeterminacy is then higher. We can verify that the upward sloping line for \( D = 1 \) and the downward sloping line for \( D = T - 1 \) always cross at the value of \( \alpha = \Phi \) for any value of \( \phi_w \). For the value of capital externalities \( \alpha_0(> \Phi) \), higher minimum values of labor externalities \( \beta \) are needed for given values of \( \Psi \), as the labor income tax schedule \( \phi_w \) is more procyclical or \( \phi_w \) is less countercyclical. See Figure 3. This government tax policy then makes agents’ belief-driven endogenous fluctuations less likely.

### 4.2 Cyclical capital income taxes: \( \phi_w = 0 \) and \( \phi_r < 1 \)

Section 4-2 considers the case where tax rate variability only applies to capital income. Let us now investigate how the local dynamics are affected by changes in the cyclicality of the capital income tax rate. The analytical effects of increases in \( \phi_r \) on the indeterminacy region are shown in Figures 4 and 5. It is ambiguous whether the region of indeterminacy expands or contracts. However, the dynamic effects of the cyclical capital taxes can be clearly understood, depending upon the size of the capital externalities. Before describing the consequences, let us define the following.

\(^8\)Barinci and Cheron (2001) conduct numerical simulations by setting the parameters at \( \Psi = 1 \), \( \gamma = 0.95 \), \( \delta = 0.1 \) and \( \alpha = 0.3 \). Then, \( \Phi = 0.126 \) is obtained and thus empirical evidence clearly verifies that \( \alpha > \Phi \).
Definition 1 \( \hat{\alpha}_p \) is defined as the value of \( \alpha \) corresponding to the intersection of the straight lines, \( (1 - \gamma (1 - \delta)) \alpha + \gamma (1 - \delta) = \frac{\beta}{\Psi} \) for \( D = 1 \) and \( (1 - \phi_r) \alpha + \frac{\beta}{\Psi} = 1 \) for \( D = T - 1 \). See Figures 5 and 6.

Figure 4 (5) shows that increases in \( \phi_r \) increase (decrease) the minimum values of labor externalities for any given size of capital externalities lower (higher) than \( \hat{\alpha}_p \left[ \equiv \frac{1 - \gamma (1 - \delta)}{2 - \gamma (1 - \delta)} \cdot \frac{1}{1 - \phi_r} \right] \).

Moreover, we can clarify that:

\[
\hat{\alpha}_p \to +\infty \ (\Phi) \quad \text{as} \quad \phi_r \to 1(0),
\]

where (16) means that \( \alpha_0 < \hat{\alpha}_p \ (\alpha_0 > \hat{\alpha}_p) \) is satisfied as \( \phi_r \to 1(0) \), because \( \alpha_0 \in (\Phi, 1) \) from Assumptions 3 and 5. Noting that \( \hat{\alpha}_p \) is positively related to \( \phi_r \), let us define the value of \( \phi_r \) as \( \phi_r^* \) such that \( \hat{\alpha}_p = \alpha_0 \) is obtained. We can verify the uniqueness of \( \phi_r^* \) and \( \phi_r^* \in (0, 1) \).

Now, let us consider how increasing \( \phi_r \) influences the minimum values of labor externalities required for indeterminacy \( \beta \) for the given values of capital externalities \( \alpha_0 \) and the elasticity of worker’s offer curve \( \Psi \). Figure 6 summarizes the dynamic consequences of cyclical capital tax rates. As \( \phi_r \) is higher in the range \((-\phi_r^*), [(\phi_r^*, 1)]\), the minimum values of \( \beta/\Psi \) are smaller [larger] and thus indeterminacy is more [less] likely. Unlike the cyclical labor income taxes, we can verify the non-monotonic relations between the cyclicality of capital income tax rate and the probability of indeterminacy.

5 Policy implication

Let us summarize the effects of the cyclicality of income tax rates on the likelihood of agents’ belief-driven aggregate fluctuations. Figure 3 illustrates that the minimum values of the labor externalities \( \beta \) are higher for a given value of \( \Psi \) as labor income taxes are more procyclical or less countercyclical. Thus, we can obtain the monotonically negative relations between the cyclicality of labor income tax rate and the possibility of indeterminacy. In contrast, Figure 6 illustrates that the lower [higher] the minimum values of \( \beta/\Psi \), the more procyclical the capital income taxes for the range \((-\phi_r^*), [(\phi_r^*, 1)]\). Unlike the labor income taxes, therefore, we can get the non-monotonic relationships between the cyclicity of capital income tax rate and the possibility of indeterminacy.

Comparing the green curve in Figure 3 and the green lines in Figure 6, we can see that the cyclicality of tax rates on labor and capital income have the same qualitative effects on the likelihood of indeterminacy only for the range \((\phi_r, 1)\), although capital and labor income taxes have the completely reversal impacts for relatively wide range of cyclicity. These analytical results imply the appropriateness of separating labor and capital income to examine how cyclical income taxes influence the likelihood of indeterminacy.

Let us clarify the theoretical correspondence of the present paper to Gokan (2008). Based on almost the same model as in this paper, Gokan (2008) analyzed endogenous income taxes that are essentially similar to countercyclical income taxes and showed that indeterminacy (divergency) is more (less) likely in endogenous labor income taxes than in
endogenous capital income taxes. This paper has already proved that the probability of indeterminacy is higher (lower) if there exist countercyclical labor (capital) income taxes. Compare the points of $\phi_w < 0$ ($\phi_r < 0$) with the point of $\phi_w = 0$ ($\phi_r = 0$) in Figure 3 (6).

Moreover, note that the upward sloping curve for $D = 1$ in Figure 3 and the downward sloping line for $D = 1$ in Figure 6 cross the vertical axis at the same point of $\beta/\Psi = [\{1 - \gamma (1 - \delta)\} \alpha_0 + \gamma (1 - \delta)]$. These figures verify that the indeterminacy (divergency) parameter region is bigger (smaller) in countercyclical labor income taxes $\phi_w < 0$ than in countercyclical capital income taxes $\phi_r < 0$. Thus, the analytical properties of countercyclical tax rates in this paper are completely compatible with in Gokan (2008). In countercyclical income taxes, we can analyze how the probability of indeterminacy is affected if the countercyclicality of income tax rates changes. In endogenous income taxes, however, we cannot investigate whether or not higher countercyclical income tax rates work as an "automatic stabilizer". More importantly, the procyclical income taxes cannot be investigated in endogenous income taxes as in the present paper.

Gokan (2008) also studied endogenous government spending in that public spending endogenously adjusts for given fixed tax rates on labor and capital income. Compare Equation (8) in the present paper with Gokan (2008, Section 7). If the variability of labor and capital income tax rates is set to zero, i.e., $\phi_w = 0$ and $\phi_r = 0$, the framework in the present paper completely coincides with the one in Gokan (2008, Section 7). From Equations (14) and (15) into which $\phi_w = 0$ and $\phi_r = 0$ are introduced, the local dynamics are independent of the tax level parameters $\eta_w$ and $\eta_r$. Therefore, the analytical property of Gokan (2008, Section 7) is equally preserved in the special case of the present paper.

Finally, we briefly state the empirical evidence about the cyclical income taxes. From the empirical study of Hall and Rabushka (1995), $|\phi_r| > |\phi_w|$ is realized in the US. If we consider procyclical income taxes, the effects of the tax variability on the probability of indeterminacy are qualitatively the same in the range of $\phi_r < 1$ and $0 < \phi_w < \phi_r$. Thus, the empirical evidence suggests that the importance of dividing capital and labor income may fall. Next, let us consider countercyclical income taxes. Defining a certain value of cyclicality as $\phi_\# \in (-\infty, 0)$, the theoretical relations are the opposite for the range $\phi_r < \phi_\# < \phi_w < 0$, and the importance of separating labor and capital income taxes holds, as in Section 4.

6 Interpretation

Here, let us provide the intuitions underlying the analytical consequences. To check the intertemporal movements of labor, we rewrite (9) and (10) as:

$$\eta_w \left(\frac{w}{l}\right)^{\phi_w} \left(b Ak^{\alpha}\mathbb{1}_{t+1} \beta\right)^{1-\phi_w} = (1 + \mu) l^\Psi_t, \quad (17)$$

$$k_{t+1} = \gamma \left[\eta_r \left(\frac{rK}{l}\right)^{\phi_r} \left(a Ak^{\alpha}\mathbb{1}_{t} \beta\right)^{1-\phi_r} + (1 - \delta) k_t\right]. \quad (18)$$

Starting from the steady-state equilibrium, a higher expected return on money shifts
the labor supply curve outward \(^9\) and thus leads to an increase in the hours of work \(l_t\) if the labor demand curve is downward sloping (i.e., \(\beta < 1\)). As shown in (18), the increase in \(l_t\) generates an increase in tomorrow’s capital stock \(k_{t+1}\) through the saving of capitalists. (17) shows that the increases in current labor \(l_t\) and future consumption \(c_{t+1} (= l_t^\phi)\) must be offset by increases in tomorrow’s after-tax wage bill [i.e., the lefthand side in (17)]. Alternatively, tomorrow’s capital stock \(k_{t+1}\) being given, a proper change in future hours \(l_{t+1}\) is needed.

As \(\phi_w\) is higher, a higher rise in future hours \(l_{t+1}\) is necessary, because the increases in the future after-tax wage bill are smaller for given increases in labor \(l_{t+1}\) and capital \(k_{t+1}\). It is then less likely for the hours worked to return to the steady state. Therefore, higher values of labor externalities \(\beta\) are needed for indeterminacy to be possible.\(^{10}\) Therefore, cyclical labor income taxes have the monotonically negative impacts on the probability of agents’ belief-driven aggregate fluctuations.

Finally, consider the case of \(\phi_r \neq 0\). For the range \((-\infty, \phi_r)\), the rise in tomorrow’s capital \(k_{t+1}\) is significantly large when the current labor \(l_t\) increases. Thus, the future labor \(l_{t+1}\) must decrease to satisfy (17). However, as \(\phi_r\) is higher in the range \((-\infty, \phi_r)\), the increases in \(k_{t+1}\) is smaller. Then, the decrease in \(l_{t+1}\) need not to be larger and thus indeterminacy is more likely. For the range \((\phi_r, 1)\), however, the rises in tomorrow’s capital \(k_{t+1}\) are not large when \(l_t\) increases. Thus, the future labor \(l_{t+1}\) must increase to satisfy (17). As \(\phi_r\) is higher in the range \((\phi_r, 1)\), the increase in \(k_{t+1}\) is smaller. Then, the increase in \(l_{t+1}\) need to be sufficiently large and thus indeterminacy is less likely. Therefore, cyclical capital income taxes have the non-monotonically qualitative effects on the occurrence of indeterminacy.

7 Progressive and regressive income taxes

Section 7 analyzes progressive and regressive income taxes where agents’ choices affect the marginal income tax rates, while the previous sections consider cyclical income taxes where income affects the marginal income tax rates, but agents regard the marginal tax rates as independent of their own actions.

In progressive and regressive income taxes, we can show that the obtained conclusions are almost the same as in Section 4. For the analytical tractability, as emphasized in Appendix A, we must assume the following:

**Assumption 6** \(\delta = 1\).\(^ {11}\)

Considering (A.16) and (A.20), (9) and (10) are respectively rewritten as:

\[
k_{t+1} = \gamma (1 - \phi_r) \eta_r (rk)^{\phi_r} (r (k_t, l_t) k_t)^{1-\phi_r},
\]

\(^9\)The labor supply curve can be obtained by combining the workers’ budget constraint and (2) to eliminate \(c_{t+1}\). By examining this labor supply curve, this property can be understood.

\(^{10}\)From (17), increases in future labor \(l_{t+1}\) are lower as the labor externalities are greater.

\(^{11}\)As the model considered in the present paper is a representative agent model but not an overlapping generations (OG) model, this assumption is imposed only for analytical tractability and is empirically implausible.
\begin{equation}
\eta_w (w l) \phi_w (w (k_{t+1}, l_{t+1})^{1-\phi_w} = \left( \frac{1}{1-\phi_w} \right)^{\frac{1}{\gamma-1}} (1 + \mu) l_t^\psi .
\end{equation}

If the progressivity of income taxes is one or more, the agents’ optimal decisions are that both the labor supply and the investment level are zero. See Appendix A. Unlike cyclical income taxes, the lower bound on the progressivity \( \phi_r \) must be imposed to obtain an interior steady state. In other words, \( \phi_r > \frac{2-\gamma}{\gamma-1} \) is required for the capitalists’ consumption to be positive \( c_t^* > 0 \). See (A-21) in Appendix A. Therefore, the following is easily obtained:

**Lemma 2** If \( \phi_w < 1 \) and \( \frac{2-\gamma}{\gamma-1} < \phi_r < 1 \), the uniqueness of positive steady state \((k^*, l^*)\) can be verified.

To examine the local dynamics, we linearize (20) and (21) near the steady state \((k^*, l^*)\) and can verify the following:

**Lemma 3** The Jacobian obtained by substituting \( \delta = 1 \) in the matrix of (11) is equivalent to the Jacobian in this section.

**Proof.** As the differences exist only in the coefficients on the right-hand sides when (20) and (21) are respectively compared with (9) and (10), lemma 3 is clearly satisfied.

Therefore, in progressive and regressive income taxes, the conclusions are almost the same as in Section 4, and progressive labor and capital income taxes also have relatively different impacts on the appearance of indeterminacy.

As mentioned above, Assumption 6 is imposed only for the analytically tractable reason. However, if we consider the case that the government imposes taxes on gross capital income \( R_t k_t \) where \( R_t \equiv r_t + (1 - \delta) \) is defined, it is not necessary to assume the 100% depreciation rate of capital as here. Then, the model can be analytically tractable and it can be clarified that the same implications in this section are obtained. This tax policy is also considered in the Working Paper version of Dromel and Pintus (2008).

### 8 Concluding remarks

The existing literature does not distinguish between labor and capital income taxes when investigating how the variability of income tax rates affects the occurrence of indeterminacy. In contrast, this paper clearly divides capital and labor income taxes, and compares how the nonlinearity of income tax rates influences the likelihood of indeterminacy.

If the variability of labor income tax rate increases, there is a contraction in the likelihood of indeterminacy. Put differently, we can verify the monotonically negative relations between the variability of labor income tax rate and the possibility of indeterminacy. However, there is an expansion in the likelihood of indeterminacy if the variability of tax rate on capital income increases toward some critical value, while there is a contraction in the likelihood of indeterminacy as the variability of capital income taxes become higher in a
range above this critical value. Unlike the labor income taxes, we can obtain the non-monotonic relations between the variability of capital income taxes and the probability of indeterminacy.

It is also proved that the effects of capital and labor income taxes on the range of externalities leading to indeterminacy are completely overturned for a sufficiently wide range of variability. The present paper implies the theoretical appropriateness of dividing income taxes into labor and capital categories when evaluating how nonlinear income taxes affect the possibility of aggregate fluctuations as driven by changes in agents’ self-fulfilling expectations. This suggests that the specification commonly adopted in the existing literature may not be sufficient when evaluating whether the nonlinearity of income taxes stabilizes the economy against agents’ belief-driven aggregate fluctuations as an "automatic stabilizer".

**Appendix A (How does the representative agent model reduce to the overlapping generations model shown in this paper?)**

Cyclical income taxes are equivalent to the case where income affects the marginal income tax rates, but agents view the marginal tax rates as independent of their own actions. In contrast, progressive income taxes correspond to the case where agents’ choices affect the marginal income tax rates. In the two types of nonlinear income taxes, Appendix A proves that the representative agent model can reduce to the overlapping generations model as shown in this paper near the steady state.

**Workers**

Identical workers face a financial market imperfection that prevents them from borrowing against after-tax labor income. Thus, workers are subject to the liquidity constraint (A–2) along with the usual budget constraint (A–1). Noting that the contemporaneous utility function of workers takes the form of $u(c_w^t) - \rho v(l_t)$, the workers’ program can be represented as follows (the superscript $w$ represents “worker” and it is assumed that the utility function satisfies the same properties as in Section 2):

$$\text{Max} \sum_{t=0}^{\infty} \rho^t [u(c_w^t) - \rho v(l_t)],$$

s.t.,

$$\frac{M_{t+1}^w - M_t^w}{p_t} + k_{t+1}^w - (1 - \delta) k_t^w = \Omega_l^w (w_l l_t) + \Omega_k^w (r_t k_t^w) - c_t^w, \quad (A-1)$$

$$k_{t+1}^w - (1 - \delta) k_t^w + c_t^w \leq \frac{M_t^w}{p_t} + \Omega_k^w (r_t k_t^w) \quad (A-2)$$

where $\rho$ denotes the discount rate on the future, and the after-tax labor and capital income are respectively specified as $\Omega_l^w (w_l l_t) = (1 - \tau_{wl}) w_l l_t$ and $\Omega_k^w (r_t k_t^w) = (1 - \tau_{rt}) r_t k_t^w$.

The first-order conditions of this problem are as follows:

$$u'(c_t^w) \geq \rho \left[ \Omega_k^w (r_{t+1} k_{t+1}^w) \right] r_{t+1} + (1 - \delta) \right] u'(c_{t+1}^w), \quad (A-3)$$

where $\Omega_k^w (r_{t+1} k_{t+1}^w) = 1 - \tau_{rt+1}$, if the capital income tax rate $\tau_{rt+1}$ is not internalized,
while \( \Omega_k^{w'}(r_{t+1}k_{t+1}^c) = (1 - \phi_r) \eta_r (\overline{r}k_{t+1})^{\phi_r} (r_{t+1}k_{t+1})^{-\phi_r} \), if \( \tau_{rt+1} \) is internalized,

\[
\Omega_t^{w'}(w_t l_t) w_t' (c_t^w) \geq \rho w' (l_t),
\]

\[
\frac{p_t}{p_{t+1}} \Omega_t^{w'}(w_t l_t) w_t' (c_{t+1}^w) \leq w' (l_t),
\]

where \( \Omega_t^{w'}(w_t l_t) = 1 - \tau_{wt} \) in cyclical labor income taxes, and \( \Omega_t^{w'}(w_t l_t) = (1 - \phi_w) \eta_w (\overline{w}l_t)^{\phi_w} (w_t l_t)^{-\phi_w} \) in progressive labor income taxes. When conditions (A–3), (A–4) and (A–5) hold with equality, \( k_{t}^{w} > 0 \) and the liquidity constraint is slack and \( M_{t}^{w} > 0 \), respectively.

**Capitalists**

For capitalists, we assume that their preferences are logarithmic and they do not supply labor to the market. They are accordingly subject only to the usual budget constraint (A–6). Their optimization problem is as follows (the superscript \( c \) represents “capitalist”):

\[
\text{Max} \quad \sum_{t=0}^{\infty} \gamma^t \ln c_t^c,
\]

s.t.,

\[
\frac{M_{t+1}^c - M_t^c}{p_t} + k_{t+1}^c - (1 - \delta) k_t^c = \Omega_k^c (r_t k_t^c) - c_t^c,
\]

where the after tax capital income is specified as \( \Omega_k^c (r_t k_t^c) = (1 - \tau_{rt}) r_t k_t^c \).

The capitalists’ first-order conditions are as follows:

\[
\frac{1}{c_t^c} \geq \gamma \left[ \Omega_k^c (r_{t+1}k_{t+1}^c) r_{t+1} + (1 - \delta) \right] \frac{1}{c_{t+1}^c},
\]

where \( \Omega_k^c (r_{t+1}k_{t+1}^c) = 1 - \tau_{rt+1} \) in cyclical capital income taxes, and \( \Omega_k^c (r_{t+1}k_{t+1}^c) = (1 - \phi_r) \eta_r (\overline{r}k)^{\phi_r} (r_{t+1}k_{t+1}^c)^{-\phi_r} \) in progressive capital income taxes,

\[
c_{t+1}^c \geq \gamma \left( \frac{p_t}{p_{t+1}} \right) c_t^c.
\]

If conditions (A–7) and (A–8) hold with equality, \( k_{t}^c > 0 \) and \( M_t^c > 0 \) can be respectively obtained.

Noting \( \gamma \in (0, 1) \), (A–8) holds with inequality in the nonzero steady states as long as the fixed rate of monetary growth \( \mu \) is not negative. Thus, \( M^{c*} = 0 \) (\( * \) indicates the steady-state value). If (A–7) also holds with inequality, \( k^{c*} = 0 \). From (A–6), then, the steady-state level of capitalists’ consumption is zero; i.e., \( c^{c*} = 0 \). Accordingly, (A–7) must be binding, and consequently \( k^{c*} > 0 \). At the steady states, we can obtain:

\[
\Omega_k^c (r^* k^{c*}) r^* + 1 - \delta = \frac{1}{\gamma}.
\]

Note the assumption \( \rho < \gamma \). From (A–9), (A–3) holds with inequality, and thus workers
choose not to hold productive capital at the steady states and nearby, i.e., \( k^w = 0 \), if:

\[
\gamma > \frac{\Omega_k' (r^* k^w)}{\Omega_k (r^* k^c)} r^* + 1 - \delta.
\]  

(A-10)

The right-hand side in (A–10) is equal to one, regardless of whether the capital income tax rate is internalized. Thus, (A–10) is satisfied.

As for (A–8), \( M^c = 0 \) implies that (A–5) must be binding \( (M^w \) must be positive) to rule out the case of \( (\frac{M}{p})^* = 0 \). Substituting (A–5) into (A–4), (A–4) holds with inequality, and accordingly the liquidity constraint is binding. This consequence does not depend upon whether the labor income tax rate is internalized.

Equilibrium

Thus, the optimal behaviors near the steady states are summarized as:

\[
\begin{align*}
\frac{p_t}{p_{t+1}} \Omega_l' (w_t l_t) w_l u' (c_{t+1}^w) &= v' (l_t), \\
\frac{p_{t+1}}{p_t} c_{t+1}^w &= \Omega_l' (w_t l_t), \quad \text{(A-11)} \\
\frac{1}{c_t^w} &= \gamma \left[ \Omega_k' (r_{t+1} k_{t+1}^c) r_{t+1} + (1 - \delta) \right] \frac{1}{c_{t+1}^c}, \\
k_{t+1}^c + c_t^w &= \Omega_k (r_t k_t^w) + (1 - \delta) k_t^c. 
\end{align*}
\]

(A-13) (A-14)

where (A–12) corresponds to the workers’ budget constraints. In cyclical income taxes, (A–11) becomes (2), while in progressive income taxes, (A–11) becomes

\[
(1 - \phi_w) (c_{t+1}^w)^{1-\sigma} = l_t^{1+\xi}. 
\]

(A-15)

If the degree of progressivity exceeds one, i.e., \( \phi_w > 1 \), agents do not supply labor to the market. When \( \phi_w < 1 \), (A–15) is written as

\[
c_{t+1}^w = \left( \frac{1}{1 - \phi_w} \right)^{\frac{1}{\sigma}} l_t^w. \quad \text{(A-16)}
\]

Next, let us derive the capitalists’ investment and consumption functions. For this purpose, by combining (A–13) and (A–14) we obtain:

\[
\begin{align*}
c_t^w \sum_{s=0}^{\infty} \gamma^s \Pi_j^{s} \left[ \frac{\Omega_k' (r_{t+j} k_{t+j}^c) r_{t+j} + (1 - \delta)}{\Omega_k (r_{t+j} k_{t+j}^c) (k_{t+j}^c)^{-1} + (1 - \delta)} \right] = \Omega_k (r_t k_t^w) + (1 - \delta) k_t^c. 
\end{align*}
\]

(A-17)

If we consider the case of cyclical income taxes, \( \frac{\Omega_k' (r_{t+j} k_{t+j}^c) r_{t+j} + (1 - \delta)}{\Omega_k (r_{t+j} k_{t+j}^c) (k_{t+j}^c)^{-1} + (1 - \delta)} = 1 \) can be ob-

\[\text{\underline{Combining (A–1) and (A–2) yields this equation.}}\]
tained. Thus, we can solve (A–17) as:

\[ k_{t+1}^c = \gamma [\Omega_k^c (r_t k_t^c) + (1 - \delta) k_t^c], \]

(A-18)

\[ c_t^c = (1 - \gamma) [\Omega_k^c (r_t k_t^c) + (1 - \delta) k_t^c]. \]

(A-19)

In contrast, if we consider the case of progressive income taxes, (A–17) is not analytically tractable. However, suppose that the rate of capital depreciation is set at \( \delta = 1 \). Then, (A-17) becomes analytically tractable and we can obtain

\[ k_{t+1}^c = \gamma (1 - \phi_r) \Omega_k^c (r_t k_t^c), \]

(A-20)

\[ c_t^c = [1 - \gamma (1 - \phi_r)] \Omega_k^c (r_t k_t^c). \]

(A-21)

(A-20) shows that investment activities are not performed if the degree of progressivity exceeds one. Although progressive income taxes are considered in Section 7-2, it is assumed that the depreciation rate of capital is set at 100% due to the analytical tractability.

References


Figure 1: The case of $w_0 = 0$ and $r_0 = 0$

Note: $\Phi = \frac{1 - \gamma (1 - \delta)}{2 - \gamma (1 - \delta)}$ and $\Xi = \frac{1}{2 - \gamma (1 - \delta)}$.

Figure 2: The case of $w_0 \neq 0$ and $r_0 = 0$

Note: As for Figures 2, 4 and 5, the indeterminacy region before (after) increasing $w_0$ is framed in by the orange (red) lines.
Figure 3: Local stability and cyclical labor income taxes

\[
\frac{\beta}{\Psi} \left[ (1 - \gamma(1 - \delta)) \alpha + \gamma(1 - \delta) \right] = (1 - \phi_c) \frac{\beta}{\Psi}
\]

Sink (locally in determinate)

Source

Saddle (locally indeterminate)

Figure 4: \( \phi_c < \phi_i < 1 \) and \( \phi_w = 0 \)
Figure 5: \( \phi_r < \phi^* \) and \( \phi_w = 0 \)

\[
(1 - \phi_r) \alpha + \frac{\beta}{\Psi} = 1
\]

\[
\gamma (1 - \delta)
\]

Sink
(locally in deterministic)

\[
[1 - \gamma (1 - \delta)] \alpha (1 - \phi_r) + \gamma (1 - \delta) = \frac{\beta}{\Psi}
\]

Source

\[
\alpha_0 (\phi_r - 1) + 1 = \frac{\beta}{\Psi}
\]

Saddle
(locally deterministic)

Figure 6: Local stability and cyclical capital income taxes