Research Note

An Essay on Mathematical Structures of the Concepts of Medium, Form, Time and System in Sociological Systems Theory

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Abstract: This paper explores the foundational concepts of sociological systems theory developed by Niklas Luhmann through the lens of axiomatic formalization, specifically focusing on the concepts of medium, form, time, and system. The approach taken involves the adoption of axiomatization within set theory, implemented in three key steps. Firstly, the extraction of an axiomatic system, termed the General Axiomatic system of Social Time (GAST), is conducted from the conceptual framework of social time formulated by P. A. Sorokin and R. K. Merton. Secondly, several distinctive axioms inherent to the fundamental concepts of Sociological Systems Theory (SST) are derived through formalization. Thirdly, a comparative analysis between the GAST and SST axiomatic systems is performed, unveiling the unique characteristics of time conception in SST. As a result, this paper arrives at three primary conclusions. 1) The method of axiomatization proves applicable in analyzing fundamental concepts in SST, encompassing medium, form, actualization, system, circumstance, self-referential reproduction, event, and time. 2) Within SST, the concept of time emerges as a specific manifestation of the broader conceptual framework of social time introduced by Sorokin & Merton. This concept can be reformulated into an axiomatic system, integrating its distinctive axioms into GAST. 3) Time conception in SST demonstrates four distinct features compared to GAST, encompassing: I) the self-referential reproduction of the system as a social fact; II) the inclusion of two layers within time, namely, time medium and time forms; III) the introduction of the concept of an event that serves as a link between the notions of system and time; and IV) the theorem asserting that self-referential systems, comprised of temporalized events, construct their own temporal structures.

Keywords: the sociology of time, social time, axiomatization, set theory, sociological systems theory, medium, Niklas Luhmann

1. Aim

This paper employs the axiomatic method, employing terminologies from set theory to elucidate the mathematical structures underlying concepts such as media, form, time, and system within Niklas

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Luhmann's sociological systems theory (**SST**: Sociological Systems Theory)[1].

2. Axiomatic Method in Sociology

T. J. Fararo, one of the most prominent sociologists who adopt axiomatics, summarizes it as follows (Fararo 2002: 168). An axiomatic system consists of 1) some *primitive terms*, which are not defined but merely presupposed, 2) *mathematical-logical terms* such as "and," "three", and 3) *primitive statements* called *axioms*, which are not demonstrated because they include only primitive and mathematical-logical terms. There are two procedures applicable to axiomatic systems. The procedure of *definition* is to define terms by primitive terms and the procedure of *deduction* is to demonstrate statements with axioms and defined terms. The method of axiomatization, therefore, has two steps: 1) description of an object (concept or theory) as axiomatic system and 2) application of the axiomatic procedures to the system in order to elucidate logical structures within it.

He also refers to "a particular mode of axiomatization that proved most useful in the social and behavioral sciences" (Fararo 2002: 170). It is called by a logician Suppes (1957) "axiomatization within set theory." Fararo summarizes the mode as follows:

> The procedure has two steps. In the first step, we specify the primitive entities as set –theoretical entities. In other words, primitive terms are given an interpretation in terms of set theory and hence mathematics. This means that one can employ such entities as matrices, functions, and the like, all of which are abstract set-theoretic entities. In the second step, the axioms are regarded as constraints on the interrelations of these entities. The axioms are construed as defining a new "set-theoretical predicate." … This procedure or something like it, has been used in a variety of theoretical contexts in the social and behavioral sciences, including formal language theory in linguistics, general equilibrium theory in economics, learning theory in psychology and balance theory in social psychology (Fararo 2002: 170).

This paper endeavors to apply the set-theoretical mode of axiomatization to concepts in sociological systems theory.

3. The General Axiomatic System of Social Time

This paper adopts the sociological concept of time formulated in Sorokin and Merton (1937) as a paradigmatic example of social time. They argue as follows:

> [S]ocial time expresses the change or movement of social phenomena in terms of other social phenomena taken as points of reference. In the course of our daily activities we often make use of this means of indicating points of time (Sorokin & Merton 1937: 618).

From this argument we can extract an axiomatic system to be called the *General Axiomatic system of Social Time* (**GAST**). It consists of the following two sets and two mappings:

The General Axiomatic system of Social Time

1. Set of time T

   $\exists\, T\, \forall\, t\, (t \in T)$

   There is a set T such that any t is an element of it.[2]

2. **Mapping of successor** $\mathrm{suc}(t) \in T$

   Any t has its successor $\mathrm{suc}(t)$ and $\mathrm{suc}(t)$ is included in T.

   The mapping of successor is surjective and hence any t has at least one element in the domain, that is, predecessor.

   2.1 $t \neq \mathrm{suc}(t)$

   Any t is not identical with $\mathrm{suc}(t)$.[3]

3. **Set of social facts** F

   $\exists\, F\, \forall\, f\, (f \in F)$

   There is a set F such that any f is an element of it.[4]

   3.1 $F = \{f_1, f_2, \cdots, f_n | n \in \mathbf{N}\}$

   F is a finite set. (**N** means the set of all natural numbers.)[5]

4. **Mapping of event** $e : F \to T \Leftrightarrow t = e(f)$

   There is a mapping e from F to T which shall be called event.[6]

All terms and statements mentioned above are primitive, that is, presupposed as undefined or unproved. Further, this system can be interpreted as functionalist one:

All time systems may be reduced to the need of providing means for synchronizing and co-ordinating the activities and observations of the constituents of groups (Sorokin & Merton 1937: 627).

The thesis this sentence directly advocates is that all systems of social time have causal relationship with, and therefore can be reduced to, the properties of the societies that adopt them (the strong functionalist view of social time). And furthermore, it can be said that this thesis supposes a more general proposition that any systems of time in a society necessarily has some interrelations with the social (the weak functionalist view of social time). The sociological conception of time to which GAST should be applied cannot merely be strong functionalist in particular but also weak functionalist in general, because the mapping of event means that if there is a social fact, then there must be the point of time that corresponds to it. In other words, the axiom of mapping of event expresses that any social fact has a relationship with a point of the social time.

4. An Axiomatization of Sociological Systems Theory

In this section fundamental concepts in SST, including the concept of time, shall be scrutinized in terms of set-theoretical type of axiomatization in order to investigate characteristics of time concept in SST and

axiomatic formulation of a proposition in SST that time is constructed in systems.

4.1 Mathematical Structures in the Concepts of Medium and Form

First, a pair of concepts of medium and form, which includes the concept of time logically, shall be formulated in terms of set theory before the concept of time shall be. Luhmann describes them as follows:

> The difference between medial substrate and form decomposes the general problem of structured complexity with the help of further difference between loosely and strictly coupled elements. This difference assumes that not every element can be linked to every other; however, it reformulates the selection problem posed by this, before addressing it, once again through an additional, preceding difference. This allows forms (in this narrower sense of strict coupling) to be represented as selections within a medium (Luhmann 1997: 196).

This conception of medium and forms can be formulated axiomatically as follows:

$(n,\ p,\ q \in \mathrm{N})$

5. **Set of medium** $\exists\, M_n \,\forall\, m_n\, (m_n \in M_n)$
   There is a set M_n such that any m_n is an element of it.

6. **Mapping of actualization** $a_p{:}M_n \rightarrow M_{n\cdot p} \subseteq M_n \Leftrightarrow m_{n\cdot p}{=}a_p(m_n)$
   There is a mapping a_p from M_n to its subset $M_{n\cdot p}$ which shall be called actualization.

7. **Set of forms** $\exists\, F_n \,\forall\, a_p\, (a_p \in F_n)$
   There is a set F_n such that any a_p whose underlying set is a set M_n is an element of it.
   And any subset of F_n is a form $F_{n\cdot q}$ of the medium M_n.

4.2 Mathematical Structures in the Concept of Time

The concept of time in SST can be interpreted as a specific case derived from general concepts of medium and form. Therefore, this section explicates mathematical structures in time concept by introducing the formulations of the concepts of medium and form described above into GAST.

Luhmann insists that time should be understood as a difference between before and after as follows:

> The dimension of time is constituted by extending the difference between before and after, which is immediately perceptible in all events, into special horizons, namely, into the past and the future. (⋯) Time is thus spanned between only its associated special horizons, marking and making the unreachable referential: between the past and the future. Therefore, time, for systems of meaning [social systems is a kind of systems of meaning: annotation by the citer], is the interpretation of reality in terms of a difference between the past and the future. In this context, the horizon of the past (and likewise, the future) is not the beginning (or the end) of time (Luhmann 1984: 116).

He also argues that time is a medium as follows:

In summary, time is a construct of the observer. The observer can only operate when something other than themselves exists simultaneously. This remains true even when something is simultaneously present but remains inaccessible to the observer in terms of operation. (⋯) This [the construction of time: annotation by the citer], like all observations, relies on various distinctions. It distinguishes the actuality that constitutes simultaneity as *the present*, using further distinction of the excluded third term, to place this present within the differences between *past* and *future*. Hence, time is constituted by a double distinction: actuality/inactuality and, in the realm of inactuality, by the distinction between past and future based on whether potential influence is still anticipated or not (Luhmann 1990: 104ff: emphasis in original).

That time is constructed in systems by observations based on the distinction between actuality and inactuality, or actuality and potentiality, as cited above, implies that time is understood as a medium in SST. In this connection, the concept of system is formulated in 4.3 before the proposition that time is constructed in systems will be discussed in 4.4.

This conception of time in SST can be formulated axiomatically as follows:

$(n,\ p,\ q \in \mathrm{N})$

8. **Set of time** $\exists\ M_t\ \forall\ t\ (t \in M_t)$

   A set of time T is a kind of medium set.

   In other words, there is a set of time medium M_t such that any t is an element of it.

9. **Mapping of successor** $suc(t){:}M_t \rightarrow M_{t \cdot suc} \subseteq M_t \Leftrightarrow m_{t \cdot suc}=\mathrm{suc}(m_t)$

   Mapping of successor $suc(t)$, which is an operation from a set of time medium M_t to its subset $M_{t \cdot suc}$ is a kind of actualization a_p.

   At this point, the mapping of successor corresponds one specific point in time (i.e., prior) to another specific point in time (i.e., subsequent). Hence, its domain can be interpreted as the past, and its range as the future[7].

10. **Set of time forms** $\exists\ F_t\ \forall\ suc(t)\ (suc(t) \in F_t)$

    A set of time forms F_t is a set of mappings of successor.

    And a time form $F_{t \cdot q}$ is a subset of F_t.

4.3 Mathematical Structures in the Concept of System

The primary axiom of SST is that "there are systems" (Luhmann 1984: 30). This axiom is presupposed in propositions connected with time as a matter of course. Time is "a construct of the observer [as system: annotation by the citer]" (Luhmann 1990: 104). It is necessary to elucidate the structure of the time construction proposition in order to comprehend completely the time conception in SST. In this section the mathematical structure of the distinction of system/circumstance, which is one of the most fundamental concepts in SST as medium/form, is to be explicated. Luhmann (1984) argues it as follows:

For the theory of self-referential systems, the circumstance is rather a prerequisite for the system's identity because identity is only possible through difference. For the theory of temporalized

autopoietic systems, the circumstance is necessary because system events cease at every moment, and further events can only be produced with the help of the difference between the system and its circumstance. The starting point for all subsequent system-theoretical research, therefore, is not an identity but a difference (Luhmann 1984: 243).

This conception of system in SST can be formulated axiomatically as follows:

$(n, p, q \in N)$

11. **Set of system** $\exists S_n \forall o_n (o_n \in S_n)$
    There is a set S_n such that any operation o_n is an element of it.

12. **Set of circumstance** $\exists C_n \forall c_n (c_n \in C_n | c_n \notin S_n)$
    There is a set C_n that is the complementary set of S_n.

13. **Mapping of system operation** $o_{n+1}: S_n \rightarrow S_n + C_n$

    The proposition of self-referential reproduction of system expressed as formula 14, which is the most characteristic in SST, is deduced from three axioms of system (formula 11), circumstance (formula 12) and operation (formula 13).

14. **Self-referential reproduction of system** $S_{n+1} = S_n + \{o_{n+1}\}$
    This means that both self-reference $S_n \rightarrow S_n$ and external reference $S_n \rightarrow C_n$ are included as operations $\{o_{n+1}\}$ in the system itself, that is, reproduce the system as S_{n+1}.

15. **Set of social facts** $\exists S_n \forall k_n (S_n = \{k_1, k_2, \cdots, k_n | n \in N\})$
    The set of social facts (in GAST) corresponds to a social system S_n and its elements are communications (k_n) in SST.
    In other words, there is a set S_n such that any k_n is an element of it.

4.4 Time Construction in Systems

The mapping of event relates SST's proposition of time construction in systems because it expresses the relationship between SST's two fundamental distinctions of system/circumstance and medium/form. Events in a system connect communication as an element of social system with time as medium. Luhmann (1997) argues it as follows:

As communication requires time to link communication to subsequent communication, this mode of operation leads to a temporal decoupling of system and circumstance. This doesn't change the fact that the system and the circumstance exist simultaneously, and this simultaneity underlies all constitutions of time. However, within the constraints thus imposed, the system must constitute an internal time that adjusts the operational pace and time perspectives of the system to its internal possibilities (Luhmann 1997: 83).

16. **Mapping of event** $e_n{:}o_n \rightarrow a_{p\cdot n} \Leftrightarrow suc(t_n)=e_n(o_n)$

There is a mapping e_n from o_n to $a_{p\cdot n}$ which shall be called event.

In other words, each communication corresponds to an actualization $a_{p\cdot n}$ as a mapping of successor $suc(t)$ and each actualization (as a mapping of successor) can be interpreted as the present for the communication corresponding to it[8].

The proposition of time construction in systems (formula 17) is deduced from and the propositions of the set of time forms (formula 10), the set of social facts (formula 15) and the mapping of event (formula 16).

17. **Time construction in systems** $\exists E_n \forall e_n (e_n \in E_n) \Leftrightarrow E_n{:}S_n \rightarrow F_t$

There is a set of mapping E_n from S_n to F_t which shall be called event set.

This proposition means that there is a set of time forms necessarily whenever a social system is constituted as a set of communicative operations.

5. Conclusion

Thus far, the axiomatic analysis of the conception of time in SST has yielded three primary conclusions.

1. The method of axiomatization can be applied to analyze fundamental concepts in SST, including medium, form, actualization, system, circumstance, self-referential reproduction, event, and time.
2. In SST, the concept of time can be construed as a specific instance of the general conceptual framework of social time introduced by Sorokin & Merton (1937). This concept can be reformulated as an axiomatic system, integrating its distinct axioms into GAST.
3. The time conception in SST exhibits four distinct characteristics when compared to GAST: I) the self-referential reproduction of the system as a social fact; II) the inclusion of two layers within time, namely, time medium and time forms; III) the introduction of the concept of an event that serves as a link between the notions of system and time; and IV) the theorem asserting that self-referential systems, comprised of temporalized events, construct their own temporal structures.

Notes

1) Section 2 and 3 are excerpted and partly modified from Takahashi (2020), a study of sociological time concept, which adopts the same methodology of axiomatization in terms of set theory.
2) In other words, this axiom means only that it is possible to determine univocally whether an element is included in T or not. It has no substantial or semantic implication.
3) In other words, any t has order relationships with the others. And it is not excluded that a predecessor can be the successor of its successor, that is, $t = suc(suc(t))$.
4) Refer to note 2.
5) In other words, there are a finite number of social facts.
6) It can be said that a f is called event when it corresponds to a certain t.
7) "With the requirement of simultaneity [between system and circumstance: annotation by the citer], it is also established that the respective present must be used as a point of difference between the future and the past. This ensures that the future and past horizons of the system and the circumstance remain integrable, thus able to merge into world horizons. (⋯) What can differentiate a system as eigen time arises from the so

selected connection, from selected future and past events." (Luhmann 1984: 254).

8) Refer to note 7.

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研究ノート

社会学的システム理論におけるメディア・形式・時間・システム概念の数理的構造をめぐる試論

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　本稿は，ニクラス・ルーマンによって展開された社会学的システム理論（SST）の基礎的概念，とりわけメディアと形式，時間，およびシステムの公理論的形式化を行う。本稿のアプローチは，集合論に基づく公理論化を採用しており，3つの段階から成る。第一に，社会的時間の一般公理系（GAST）と呼ぶべき公理系を，P. A. ソローキンと R. K. マートンによって定式化された社会的時間の概念枠組みから取り出す。第二に，社会学的システム理論の基本概念に含まれるいくつかの特徴的な公理を形式化を通して導き出す。第三に，GAST と SST の公理系の比較を行い，SST の時間概念の独自の特徴を明らかにする。以上の段階を経て，本稿は3つの主要な結論に達した。第一に，公理論化の方法は，メディアと形式，顕在化，システムと環境，自己言及的再生産，出来事，そして時間といった SST の基本概念を分析する上でも適用可能だということである。第二に，SST の時間概念はソローキンとマートンによって導入された社会的時間の概念枠組みの特殊事例として解釈でき，それは特徴的な諸公理を GAST に組み入れるかたちで再定式化できるということである。第三に，SST の時間の捉え方は GAST と比較して4点で特徴的だということであり，1点目に社会的事実としてのシステムの自己言及的再生産，2点目に時間メディアと時間形式という時間における2つの水準，3点目にシステムと時間を結びつける出来事という概念の導入，4点目に時点化された出来事から成る自己言及的システムが独自の時制構造を構成するという定理が挙げられる。

キーワード：時間の社会学，社会的時間，公理論化，集合論，社会学的システム理論，メディア，ニクラス・ルーマン