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Reconstruction of chaotic dynamics via a network of stochastic resonance neurons and its application to speech

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Abstract Recently, a great deal of attention has been paid to *stochastic resonance* as a new framework to understand sensory mechanisms of biological systems. Stochastic resonance explains important properties of sensory neurons that accurately detect weak input stimuli by using a small amount of internal noise. In particular, Collins et al. reported that a network of stochastic resonance neurons gives rise to a robust sensory function for detecting a variety of complex input signals. In this study, we investigate effectiveness of such stochastic resonance neural networks to chaotic input signals. Using the Rössler equations, we analyze the network's capability to detect chaotic dynamics. We also apply the stochastic resonance network systems to speech signals, and examine a plausibility of the stochastic resonance neural network as a possible model for the human auditory system.

Key words Stochastic resonance · Neural network · Chaos · Speech · Auditory system

Introduction

Recently, a great deal of attention has been paid to *stochastic resonance*[1] as a potential model for sensory neurons in real biological systems.[2-4] Stochastic resonance can elucidate functions of the sensory neurons in the sense that it enables neurons to detect weak input stimuli by using a small amount of internal noise. From this viewpoint, stochastic resonance and its sensory functions have been studied in relation to a variety of neuron models, such as the FitzHugh–Nagumo neuron model and the Hodgkin–Huxley neuron model.[5] Although periodic signals have been mainly considered as the input stimuli, Collins et al.[6] extended the idea to aperiodic input signals, and demonstrated a neuronal capability to detect weak aperiodic input stimuli. In particular, they showed that a network of stochastic resonance neurons gives rise to a sensory function which is much more robust than a single neuron in the sense that the network does not require any careful tuning of the optimal noise intensity.[7] Their results therefore imply that the network structure might be of significant importance in the study of sensory functions in real neural systems.

On the basis of this work,[7] we use a network of stochastic resonance neurons as a sensory model to detect chaotic input. We study how the dynamic structure of chaos is encoded into the firing rate of the neural network. This study is of potential importance because chaotic dynamics have recently been discovered in a variety of biological systems,[8,9] and it is an important, yet unresolved, problem to consider how the dynamic characteristics of chaos are encoded into neuronal information processing systems. Although there are some related works which discuss the problems of encoding chaotic dynamics into single-neuron models,[10-12] to our knowledge, encoding chaos into neural network models has not yet been thoroughly investigated.

By using the Rössler equations as a typical example of deterministic chaos, we study response characteristics of a network of stochastic resonance neurons to chaotic input stimuli. Our main focus is on whether the geometric structure of chaotic dynamics can be encoded into a delay-coordinate of the firing rate of a neural network. By using the nonlinear prediction technique, accuracy of the geometric encoding of chaos is evaluated. Dependence of the accuracy of the geometric encoding on neural network parameters such as the noise intensity, the number of neural elements, and the time-constant parameter are also investigated. As a possible application, we further apply the neural network model to reconstruct a speech signal of a normal phonation of vowel /a/, and examine plausibility of using the

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neural network as a real physiological model for the human auditory system.

This paper is organized as follows. In the next section we introduce a network of noisy FitzHugh–Nagumo neurons as a stochastic resonance network model. We then study the response characteristics of the neural network to chaotic input generated from the Rössler equations, and investigate the network's capability to reconstruct chaotic dynamics by varying the noise intensity, the network size, and the time-constant parameter. In the following section we apply the network model to speech signals, and study how many neurons are required to reconstruct speech signals which are clear enough to be perceived by the human ear. The final section is devoted to our conclusions and discussions.

A network of FitzHugh–Nagumo neurons

Let us consider a network of FitzHugh–Nagumo (FHN) neurons[13,14] described below (see also Fig. 1).

$$\tau\varepsilon\dot{v}_i = -v_i(v_i - 0.5)(v_i - 1) - w_i + S(t) + \xi_i(t) \qquad (1)$$

$$\tau\dot{\omega}_i = v_i - \omega_i - e \qquad (i = 1, \ldots, K) \qquad (2)$$

The variables v_i and w_i stand for the dynamic states of the i-th neuron, ξ_i represents independent *Gaussian* white noise satisfying $E[\xi_i(t)] = 0$ and $E[\xi_i(t)\xi_j(s)] = 2D\delta(t-s)\delta(i-j)$, ($E[\cdot]$: ensemble average), K represents a number of neurons in the network, and e and ε represent system parameters of the FHN model, which are fixed as $(e, \varepsilon) = (0.15, 0.005)$. The time-constant parameter τ that controls the time-scale of the neural dynamics is set to be $\tau = 0.01$. We consider that the neurons of the network are uniform and commonly receive a weak *subthreshold* input $S(t)$. By subthreshold, we mean that no neuron fires without any noise. In the following numerical experiments, the dynamics of the stochastic differential Eqs. 1 and 2 are simulated by integrating the equations with a first-order approximate algorithm[15] with an integration step of $\Delta t = 5 \cdot 10^{-5}$.

When the i-th neuron potential $v_i(t)$ crosses a threshold value of $v_{th} = 0.7$, we say that the i-th neuron fires. Then the firing rate $R_i(t)$ of the i-th neuron is computed by counting the number of firing times within a duration of W. The activity of the FHN neural network is finally measured by the firing rate of the summing unit $R_\Sigma(t)$ (Fig. 1) that sums and averages the firing rates of all the neural elements as $R_\Sigma(t) = \frac{1}{K}\sum_{i=1}^{K} R_i(t)$.

Figure 2 shows response characteristics of a single neuron ($K = 1$) to the time-constant input stimuli S. The firing rate R over a long-term duration $W = 500$ is computed by increasing the input stimuli S, where the noise intensity is set as $D = 0$ and 10^{-8}. In the case of no noise ($D = 0$), the FHN model has a stable equilibrium point which corresponds to the resting state, and which never fires for a small input signal S. Hence, the firing rate is zero until $S < 0.11231$. As the input signal is increased to over $S \approx 0.11231$, a limit-cycle attractor is generated via a Hopf bifurcation of the equilibrium point, and the neuron exhibits oscillatory dynamics. The firing rate jumps up at the bifurcation point, and then monotonically increases as the input stimuli S is further increased. In contrast to the noise-free case, when there is noise of $D = 10^{-8}$, the neuron fires also in the subthreshold regime ($S < 0.11231$), and the firing rate monotonically and continuously increases as the subthreshold input S is increased. This corresponds to the stochastic resonance regime, where the low level noise enables a neuron to detect a weak input stimuli. In order to study a function of this stochastic resonance noise, in the following sections, we consider only the subthreshold input signals.

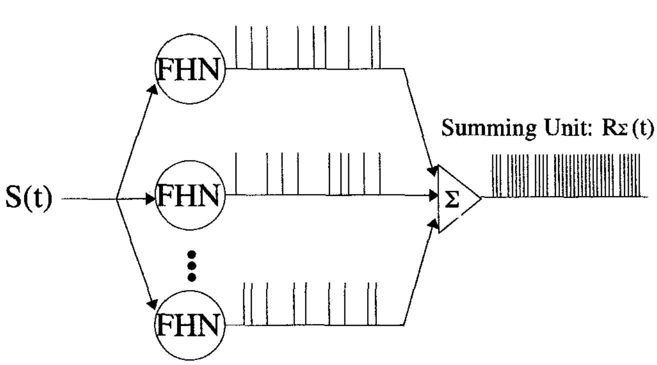

Fig. 1. Schematic illustration of a network of FitzHugh–Nagumo (*FHN*) neurons with Gaussian white noise. Each neuron received a same input stimuli $S(t)$. The network activity is measured by the firing rate of the summing unit $R_\Sigma(t)$ that computes an averaged firing rate of all the neural elements

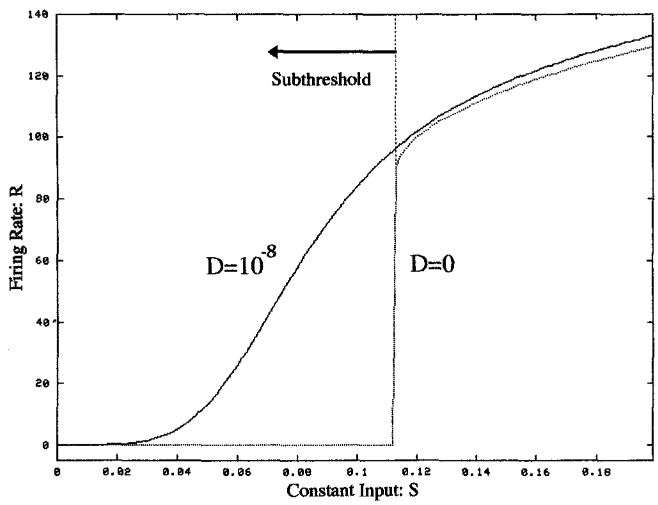

Fig. 2. Response characteristics of a single neuron to a time-constant input S. The firing rate R over a long-term duration $W = 500$ is computed by increasing the input stimuli from $S = 0$ to $S = 0.2$. The noise intensity is set to be $D = 0$ and 10^{-8}

Reconstructing chaotic dynamics

Reconstructing the Rössler attractor

In this section, we study response characteristics of the network of FHN neurons activated by chaotic input stimuli. This work presents a different viewpoint from some related works.[7,12] Castro and Sauer[12] studied the response characteristics of a single noisy FHN model to chaotic input, and reported that stochastic resonance phenomena had been observed. Collins et al.,[6,7] on the other hand, presented a theory of a network of stochastic resonance neurons, and showed the advantage of a network model that realizes a robust detection of weak input stimuli without any careful tuning of the noise intensity. In this study, we focus on chaotic input signals rather than general aperiodic signals so that we can apply techniques of nonlinear deterministic prediction to evaluate the response characteristics of the neural network.

By using the $x(t)$-variable of the Rössler equations[16]

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= bx + z(x - c) \end{aligned} \tag{3}$$

we study how the dynamic structure of the chaotic input can be encoded into the firing rate of the network of FHN neurons. Parameter values of the Rössler equations are fixed as $(a, b, c) = (0.36, 0.4, 4.5)$, and the input signal is set to be

$$S(t) = 0.05 + 0.06 \frac{x(t)}{\max_t |x(t)|} \tag{4}$$

Figures 3b,c show the dynamic structures of the chaotic input signal reconstructed by using a delay coordinate of the mean firing rate $R_\Sigma(t)$ of the FHN network as

$$\{R_\Sigma(t), R_\Sigma(t - \theta), \ldots, R_\Sigma(t - (d - 1)\theta)\} \tag{5}$$

where d and θ stand for the reconstruction dimension and the time lag, respectively.[17,18] Compared with the original chaotic dynamics in Fig. 3a, the dynamic characteristics of the Rössler attractor with "stretching" and "folding" are well reproduced in Fig. 3b,c. Although the reconstructed dynamics by a network of 20 neurons looks rather noisy in Fig. 3b, a smoother nonlinear dynamics can be reconstructed by a network of 1000 neurons, as in Fig. 3c. This is due to the effect of the network structure, which realizes a reliable and accurate detection of a weak input stimuli by using an ensemble average of the firing rates of many neural elements.

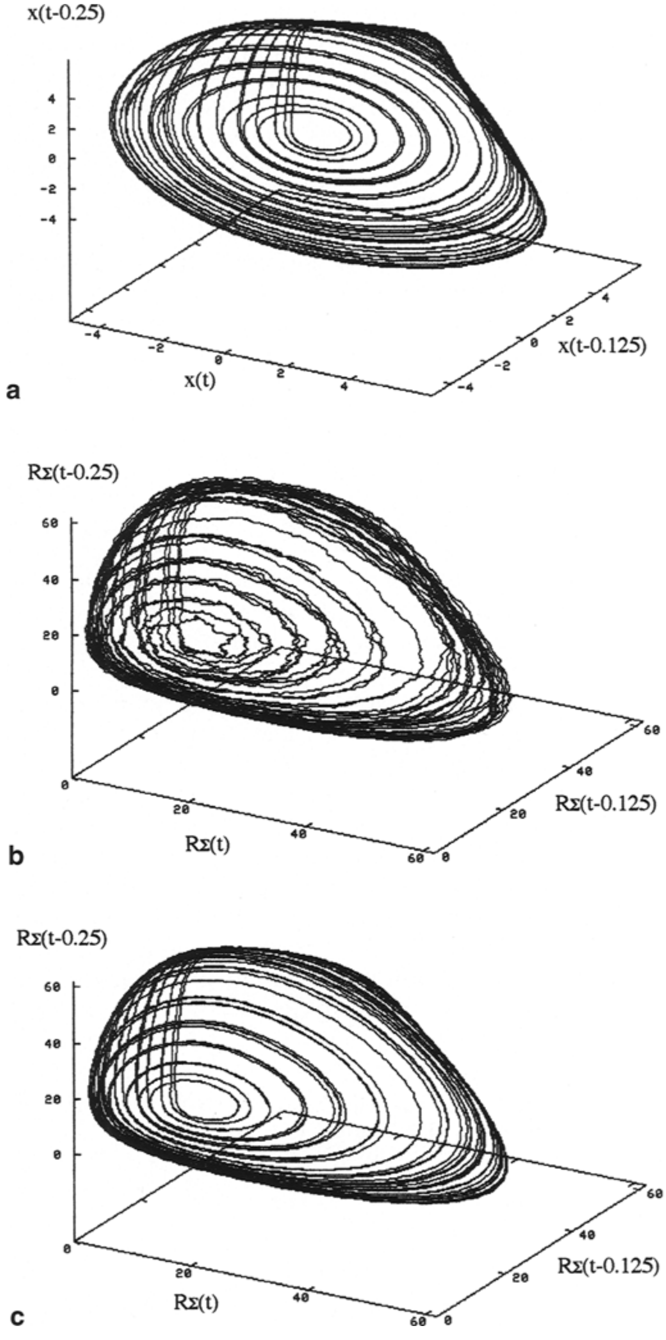

Fig. 3. a Three-dimensional delay-coordinate reconstruction $\{x(t), x(t - 0.125), x(t - 0.25)\}$ of the Rössler attractor. **b, c** Three-dimensional delay-coordinate $\{R_\Sigma(t), R_\Sigma(t - 0.125), R_\Sigma(t - 0.25)\}$ of the mean firing rate $R_\Sigma(t)$ of the network of FHN neurons with $K = 20$ and 1000, respectively

Nonlinear prediction

In this subsection, we quantify the accuracy of reconstructing chaotic dynamics by the stochastic resonance neural networks. Although Collins et al.[7] used the correlation coefficient to measure the accuracy of the signal detection, we exploit the nonlinear prediction error (NPE)[19,20] to evaluate the accuracy of the geometric reconstruction of chaotic dynamics.

First, we compute a time-series of the average firing rate of the neural network

$$\{u(n) = R_\Sigma(n\Delta) : n = 0, 1, \ldots, N\} \tag{6}$$

with a sampling rate of $\Delta = 0.1$. Then the time-series $\{u(n)\}$ is divided into first and second halves. From the first-half data, a nonlinear predictor $\tilde{f}:R^d \to R^d$, which approximates the data dynamics as $\boldsymbol{u}(n + 1) \approx \tilde{f}\,(\boldsymbol{u}(n))$, is constructed using the delay coordinate $\boldsymbol{u}(n) = \{u(n), u(n - \theta), \ldots, u(n - (d - 1)\theta)\}$. For the nonlinear predictor $\tilde{f}$, Sugihara–May's local linear predictor[20] is used. For the latter-half data, a nonlinear prediction is carried out. The forecasting procedure is that for a give initial state $\boldsymbol{u}(n)$, the p-step further state $\boldsymbol{u}(n + p)$ is predicted to be $\tilde{\boldsymbol{u}}(n + p) = \tilde{f}^p(\boldsymbol{u}(n))$ using the p-iterate of the predictor $\tilde{f}$. The NPE is finally computed as the normalized root-mean-square error

$$\mathrm{NPE} = \frac{\sqrt{\frac{2}{N}\sum_{n=N/2}^{N}\{u(n) - \tilde{u}(n)\}^2}}{\sqrt{\frac{2}{N}\sum_{n=N/2}^{N}\{u(n) - E[u]\}^2}} \qquad (7)$$

In our analysis, the reconstruction dimension d, the time lag θ, the prediction step p, and the number of the data N are set as $(d, \theta, p, N) = (3, 15, 2, 5000)$.

Figure 4 shows the nonlinear prediction curve computed for the FHN network with an increasing noise intensity from $D = 2.5 \cdot 10^{-10}$ to $5.12 \cdot 10^{-7}$. The four prediction curves correspond to networks with different sizes of K (i.e., 1, 10, 100, and 1000). First, we focus on a single neuron's response curve ($K = 1$). In a small noise regime, a very large prediction error is initially observed. As the noise intensity is slightly increased, the prediction error is significantly decreased, and the response curve produces a single sharp minimum at $D \approx 10^{-8}$. As the noise intensity is further increased, the prediction error is eventually increased. This is a typical characteristic of stochastic resonance, which is known to exhibit a small effective noise for detecting weak input signals.

Next, let use see the the resonance characteristics of the network of FHN neurons ($K = 10, 100, 1000$). We recognize a basically similar prediction curve to that of the single neuron model ($K = 1$) in Fig. 4. A significant difference is observed as range of effective noise that realizes an accurate reconstruction of chaotic dynamics. Namely, as the number of the neurons is increased, the effective noise range is widen considerably, and a robust reconstruction of chaotic dynamics is realized in a wide area of noise. As has been pointed out by Collins et al.,[7] this is due to the advantage of the network structure, which reliably estimates the strength of the input stimuli by using an ensemble averaged firing rate of many neurons. In a statistical sense, it is reasonable that the estimate becomes more accurate as the number of the neurons is increased.

Effect of the network size and the time-constant parameter

In this subsection, we study dependence of the network capability of reconstructing chaotic dynamics on the network size K and the time-constant parameter τ. By using the same chaotic input (Eq. 4) from the Rössler equations (Eq. 3), the nonlinear predictability of the chaotic dynamics reconstructed by the network of FHN neurons is computed by changing the network size and the time-constant parameter within the range of $(K, \tau) \in [10, 200] \times [0.01, 0.1]$. As shown in Fig. 5, the prediction error is decreased not only by increasing the network size K, but also by decreasing the time-constant parameter τ. The reason why the nonlinear predictability is improved by decreasing the time constant

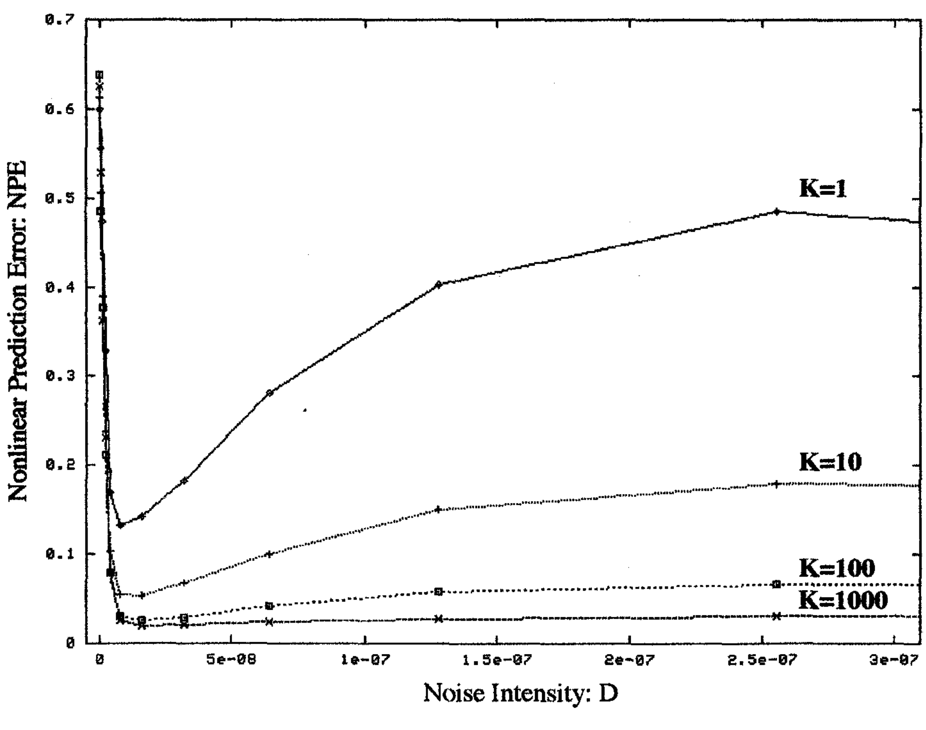

Fig. 4. Nonlinear prediction curve computed for a network of FHN neurons. The noise intensity is increased from $D = 2.5 \cdot 10^{-10}$ to $D = 5.12 \cdot 10^{-7}$, and the network size is varied as $K = 1$, 10, 100, and 1000

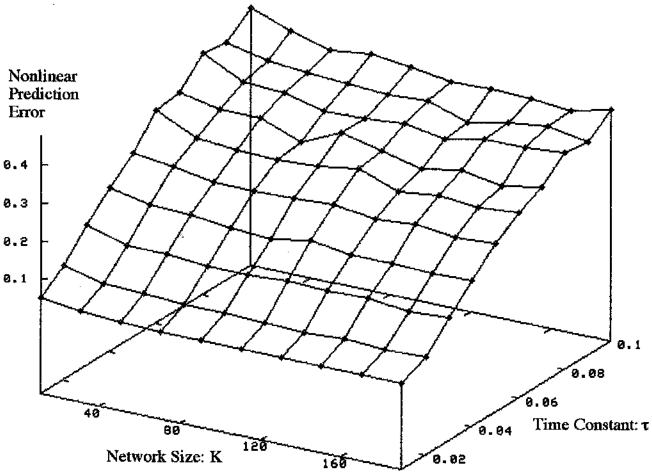

Fig. 5. Nonlinear prediction error (NPE) computed by changing the network size K and the time-constant parameter τ within the range $(K, \tau) \in [10, 200] \times [0.01, 0.1]$

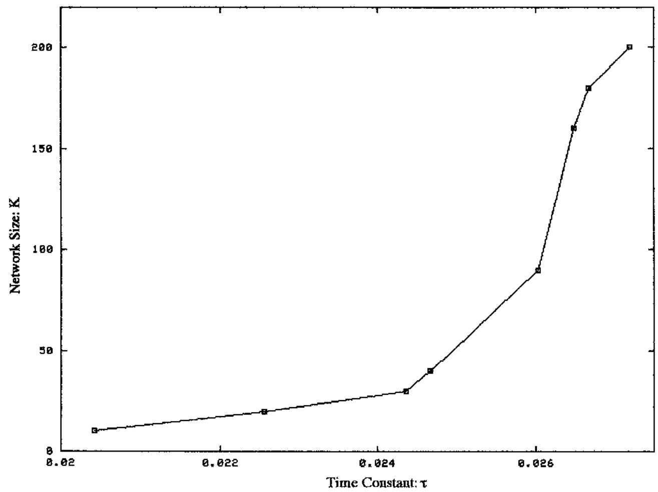

Fig. 6. Number of neurons required to reconstruct chaotic dynamics with a prediction accuracy of NPE < 0.1. The time-constant parameter is ranged as $0.02 < \tau < 0.027$

parameter can be explained as follows. As the time-constant parameter is decreased, dynamics of the FHN neurons become faster. Compared with the accelerated neural dynamics, temporal change in the input signal becomes relatively slow. As a consequence, the neuronal capability to estimate the strength of the slow-varying input stimuli is improved, and an accurate reconstruction of chaotic dynamics is realized.

Suppose we have a noisy reconstruction of chaotic dynamics by the present network model. How can we improve the reconstruction capability? As we have seen, one way is to set the time-constant value small so that the temporal resolution of a single neuron becomes high. The other way is to increase the network size so that a more reliable estimate of the input stimuli is realized by using the ensemble averaged firing rate of many neurons. From a physiological viewpoint, it is not realistic to control the network capability by changing the time-constant parameter, because biological neurons have their own physical properties which are not easy to adjust, although this is not impossible.[21] It is physiologically more plausible to control the network size in order to improve the network capability to reconstruct chaotic dynamics.

In Fig. 6, number of the neurons required to reconstruct chaotic dynamics with an accuracy of NPE < 0.1 is drawn by changing the time-constant parameter. As the time-constant parameter is increased, the number of the neurons required increases in an exponential manner. In physiological modeling of robust sensory systems, this figure presents an abstract idea of how many neurons are required for the network model.

Application to speech

In this section, we apply the network of FHN neurons to speech signals. In this experiment, we consider the neural network as a possible model for the human auditory system.

As a sample speech signal, normal phonation of a vowel /a/ (mausy003.ad) in the standard Advanced Telecommunications Research Institute International (ATR) data-base is exploited. The waveform structure of the speech signal

$$\{x(t) : 0 < t < 200\,\text{msec}\} \tag{8}$$

is shown in Fig. 7a. The subject is a male speaker who has no evidence of laryngeal pathology. The speech signal is low-pass-filtered with a cut-off frequency of 8 kHz, and digitized with a sampling rate of 20 kHz and with 16-bit resolution. The recording conditions and the speech quality are good enough in the sense that a natural vocal sound can be reproduced by D/A conversion of the speech signal.

By using the speech signal, the input signal for the neural network is set to be

$$S(t) = 0.1 + 0.01\frac{x(t)}{\max_t |x(t)|} \tag{9}$$

so that the speech is treated as a subthreshold input signal. The parameter values of the FHN neuron are fixed as $(D, W, \tau) = (5.7 \cdot 10^{-11}, 5 \cdot 10^{-5}, 0.001)$. With these parameter values, the firing rate of a single neuron activated by a constant input of $S = 0.1$ becomes 98.8 Hz. According to Kelly,[22] the average frequency of the sensory neurons in the active state in the human auditory system is about 100 Hz. Hence, this parameter setting may well correspond to the situation of real sensory neurons.

Figure 7b,c show a time-series of the firing rate $R_\Sigma(t)$ of the network of FHN neurons with $K = 10^4$ and 10^6. Although the network of 10^4 neurons generates rather noisy dynamics, the firing rate $R_\Sigma(t)$ of the network of 10^4 neurons produces a qualitatively similar time–waveform structure to the original speech signal in Fig. 7a. Figure 8 shows the nonlinear prediction errors computed for the speech signals

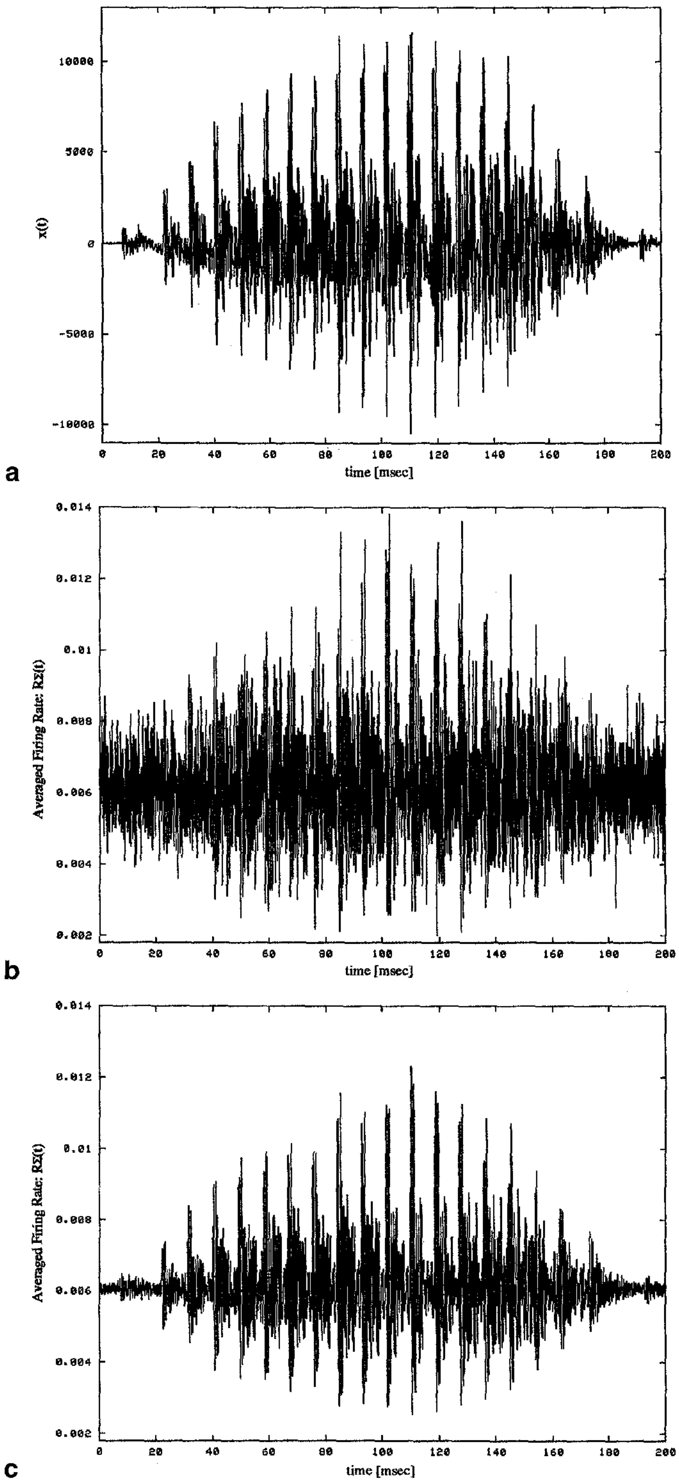

Fig. 7. **a** Speech signal of a normal phonation of the vowel /a/. **b**, **c** Time-series of the average firing rate $R_\Sigma(t)$ of the network of FHN neurons activated by the speech signal in **a**. The number of the neurons was set as $K = 10^4$ (**b**) and $K = 10^6$ (**c**)

reconstructed by the FHN neural networks with $K \in [10^4, 10^6]$. We can see that as the number of the neurons is increased, an inverse of the prediction error (1/NPE) also increases, and the accuracy of the speech reconstruction is improved.

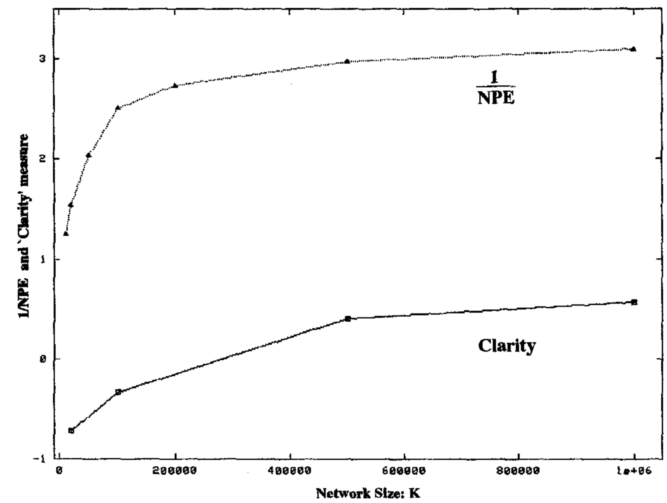

Fig. 8. Simultaneous plots of the inverse of the nonlinear prediction error (1/NPE, *dotted line*) and the "clarity" measure (*solid line*) computed for speech signals reconstructed by the FHN neural network. The network size was varied from $K = 10^4$ to $K = 10^6$

We also examined sound quality of the reconstructed speech signals on the basis of a psychoacoustic experiment. For the psychoacoustic experiment, a paired-comparison test[23] was used. For four speech signals reconstructed by the network with $K = 2 \cdot 10^4$, 10^5, $5 \cdot 10^5$, and 10^6, "clarity" of the speech sound was evaluated as follows.

Among the four speech signals, pick a pair of signals, say A and B, and synthesize the sounds one by one. Subjects are requested to judge which one is clearer than the other after listening to two signals in each paired-comparison test. The paired-comparison test was carried out for all possible pairs of the four speech signals. Using Thurstone's method,[23] the clarity of the four speech signals was finally evaluated. The subjects were consisted of ten males and ten females, where none were experts in psychoacoustic experiments.

Figure 8 shows the clarity measures of the four speech signals. As the number of the neurons is increased from $K = 2 \cdot 10^4$ to $K = 5 \cdot 10^5$, we see that the clarity of the speech improves significantly. However, only a little difference is recognized between $K = 5 \cdot 10^5$ and $K = 5 \cdot 10^6$. This implies that a network of more than $5 \cdot 10^5$ neurons is capable of reconstructing speech signals which are clear enough to be perceived as a human speech signal. This result suggests that in order to apply the stochastic resonance neural networks to auditory systems, at least $5 \cdot 10^5$ sensory neurons are required.

We also note that there is a good agreement between the inverse of the nonlinear prediction error (1/NPE, dotted line) and the clarity measure (solid line) in Fig. 8. This implies the possibility of using the nonlinear predictability as a measure for quantifying the clarity and the dynamic characteristics of speech signals. Recently, there has been increasing interest in nonlinear analysis of speech signals, and many researchers have considered possible applications of nonlinear dynamic statistics to characterize speech signals.[24–27] The present results encourage further applications of nonlinear statistics, such as nonlinear predictability, for evaluating speech signals.

Conclusions and discussions

We studied a network of FHN neurons to detect chaotic dynamics in weak input stimuli. We investigated whether the geometric structure of chaotic dynamics can be reconstructed in the delay-coordinate space of the averaged firing rate of the neural network, where the reconstruction accuracy was evaluated by nonlinear prediction errors.

Using the Rössler equations, we first studied the dependence of the network capability to reconstruct chaotic dynamics on the noise intensity. Our analyses showed a stochastic resonance property that gives rise to an intermediate noise level which efficiently reconstructs chaotic dynamics. We also showed that a more robust and accurate reconstruction of chaos can be realized by increasing the network size and by decreasing the time-constant parameter. From a physiological viewpoint, it is not realistic to change the time-constant parameter in order to improve the network's capability because real neurons have their own physical properties. It may be biologically more plausible to increase the network size in order to realize a robust sensory mechanism.

On the basis of the experiments with Rössler equations, we further applied the network of FHN neurons to reconstruct speech signals, and considered the stochastic resonance neural network as a possible model for human auditory system. By using the neural elements with a dynamic frequency of about 100 Hz, we have confirmed that a speech signal which is perceived to be clear enough for the human ear can be reconstructed by using a network of more than $5 \cdot 10^5$ neurons. This implies a possibility that the human auditory system uses this type of noisy neural network for detecting speech signals. As a possible engineering application, present result also encourages a use of the stochastic resonance network model as a sound amplifier device to aid hearing disabilities.[28]

Note that the present investigation is only preliminary in the sense that no detailed physiological structure has been considered for the human auditory system. The real auditory system has a more complex and systematic organization, such as the spectral decomposition of the basilar membrane.[22] Also, neural elements may not be as uniform as we have considered them to be in the present study. In other words, individual neurons may have different frequency ranges, and the noise intensities may be different in the brain area. Hence, an important future project is to study a network of nonuniform neuron models. More realistic auditory models should be constructed to assess plausibility of the stochastic resonance neural network for real auditory systems.

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