$\lim_{n\to-\infty}\left(1+\frac{1}{n}\right)^n = e$ を証明せよ.

解答. $m=-n$ と置き,

$$\lim_{m\to\infty}\left(1-\frac{1}{m}\right)^{-m} = e$$

を示せばよい.

$$\left(1-\frac{1}{m}\right)^{-m} = \left(\frac{m-1}{m}\right)^{-m} = \left(\frac{m}{m-1}\right)^{m} = \left(1+\frac{1}{m-1}\right)^{m-1}\left(1+\frac{1}{m-1}\right)$$

だから,

$$\lim_{m\to\infty}\left(1-\frac{1}{m}\right)^{-m} = \lim_{m\to\infty}\left(1+\frac{1}{m-1}\right)^{m-1}\lim_{m\to\infty}\left(1+\frac{1}{m-1}\right) = e\cdot 1 = e.$$