

# Material Dependence of Multiaxial Low Cycle Fatigue Damage under Non-proportional Loading

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**ABSTRACT.** *This study discusses evaluation of material dependence of multiaxial low cycle fatigue (LCF) to develop a suitable strain parameter for life estimation under non-proportional loading. It has been reported that fatigue lives are reduced accompanying an additional hardening under strain controlled non-proportional loading in which principal directions of stress and strain are changed in a cycle. Strain controlled multiaxial LCF tests using proportional and non-proportional strain paths were carried out using hollow cylinder specimens of several materials. The reduction in low cycle fatigue life due to non-proportional loading is discussed relating to the additional cyclic hardening behaviors and its material dependence. Material constant,  $\alpha$ , used in strain parameter for life estimation under non-proportional multiaxial LCF is also discussed.*

## INTRODUCTION

Components and structures like pressure vessels and high temperature exchangers undergo low cycle fatigue (LCF) damage. In multiaxial LCF under strain controlled non-proportional loading in which principal directions of stress and strain are changed in a cycle, it has been reported that fatigue lives are reduced accompanying with an additional hardening which depends on both strain paths and materials [1-5]. Thus, developing an appropriate design parameter for multiaxial LCF is required for the reliable designs and maintenances of structure components.

Itoh *et al.* [4-7] have carried out a series of multiaxial LCF tests under non-proportional loading with various strain paths combined axial and shear loadings using a hollow cylinder specimen and have examined the dependence of the life on the strain path and the material. They also proposed a strain parameter for estimating multiaxial LCF life under non-proportional loading. However, these studies were performed mainly for materials of which crystal structure is face-centered cubic lattice (FCC), but no study for materials with other crystal structures like body-centered cubic lattice (BCC). Thus, it should be necessary to examine the fatigue life properties for the

Table 1. List of materials and parameters employed.

Material	CS	$A$	$B$	$\alpha$	$\alpha^*$	$\sigma_B$	$\sigma_{0.2}$	$(\sigma_B - \sigma_{0.2})/\sigma_B$
SUS316	FCC	0.010	0.93	0.75	0.75	575	260	0.55
SUS304		0.012	0.87	0.9	0.8	750	290	0.61
SUS304 (923K)		0.011	0.14	0.40	0.52	480	130	0.73
SUS310S		0.009	0.92	0.76	0.70	520	215	0.59
OFHC		0.009	0.16	0.16	0.20	240	182	0.24
6061Al		0.018	0.16	0.41	0.48	390	253	0.35
SGV410	BCC	0.008	0.85	0.39	0.85	470	275	0.41
S25C		0.008	0.49	0.28	0.65	493	354	0.28
S45C		0.011	0.78	0.22	0.50	630	445	0.29
S55C		0.012	0.48	0.25	0.45	695	485	0.25
SUS430		0.009	0.68	0.28	0.65	480	263	0.45

materials which show the different deformation behaviors [10] and to discuss the applicability of the strain parameter for life estimation.

In this study, multiaxial LCF tests under strain controlled proportional and non-proportional loading were carried out using hollow cylinder specimens of several kinds of materials to examine the relationship between additional hardening and reduction in failure life due to non-proportional loading. This study also discusses the material constant,  $\alpha$ , used in the strain parameter for life estimation under non-proportional multiaxial LCF and proposes a simple method to reevaluate  $\alpha$  using material constant resulted from monotonic tension test.

## MATERIALS AND TEST PROCEDURE

Test materials employed were various metallic materials of which crystal structures are face-centered cubic lattice (FCC) or body-centered cubic lattice (BCC) as listed in Table 1. In the table, coefficients  $A$  and  $B$ , parameters  $\alpha$  and  $\alpha^*$ , etc. will be mentioned later. The specimen used was a hollow cylinder specimen with O.D. 12mm, I.D. 9mm and G.L. 7mm as shown in Fig. 1.

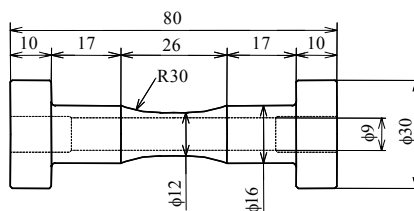


Figure1. Shape and dimensions of specimen (mm).

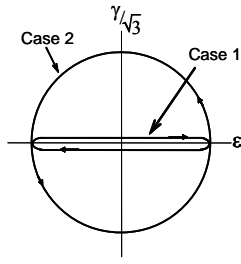


Figure 2. Strain paths employed.

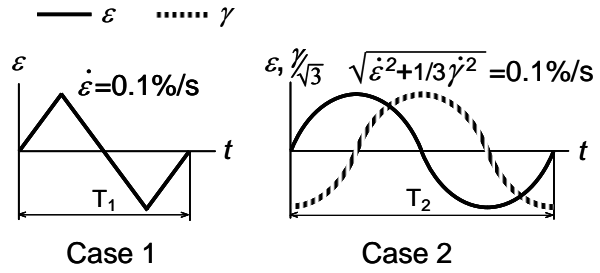


Figure 3. Axial and shear strain waveforms.

Total strain controlled multiaxial LCF tests were conducted under 2 types of strain paths. Figures 2 and 3 show the strain paths on  $\varepsilon-\gamma/\sqrt{3}$  plot and the strain waveforms of  $\varepsilon$  and  $\gamma$ , respectively, where  $\varepsilon$  and  $\gamma$  are axial and shear strains. Case 1 is the push-pull test and Case 2 the  $90^\circ$  sinusoidal out-of-phase loading test. The former is the proportional loading test and the later the non-proportional loading test. Total strain ranges were set to the same ranges in Case 1 and Case 2 and strain rate was 0.1%/sec based on Mises basis. Number of cycles to failure (failure life),  $N_f$ , is determined as the cycles at which axial or shear stress range was reduced to 3/4 from that at  $1/2N_f$ .

## RESULTS AND DISCUSSIONS

### *Strain Parameter for Life Estimation under Non-proportional Loading*

In multiaxial LCF of austenite stainless steels, failure lives decrease drastically under non-proportional loading accompanied a large additional hardening if the data are correlated by Mises' total equivalent strain range. It has been reported that the large reduction in life has a close relation with strain paths and materials. Therefore, Itoh *et al.* [4-7] proposed the non-proportional strain range for life estimation under non-proportional loading as equated,

$$\Delta\varepsilon_{NP} = (1 + \alpha f_{NP}) \Delta\varepsilon_I \quad (1)$$

where  $\Delta\varepsilon_I$  is the maximum principal strain range under non-proportional loading which can be calculated by  $\varepsilon$  and  $\gamma$ .  $\alpha$  and  $f_{NP}$  are the material constant and non-proportional factor, respectively. The former is the parameter related to the additional hardening due to non-proportional loading and the later is the parameter expressing the intensity of non-proportional loading.

The value of  $\alpha$  can be given by two methods [10]. One method is to define  $\alpha$  as the ratio of increase in stress amplitude in Case 2 to that in Case 1. The other method is to define  $\alpha$  as  $N_f$  in Case 2 becomes the life equivalent to  $N_f$  in Case 1 at the same  $\Delta\varepsilon_I$ .  $f_{NP}$  is defined as,

$$f_{NP} = \frac{k}{T \varepsilon_{I_{max}}} \int_0^T (|\sin(\xi(t))| \varepsilon I(t)) dt, \quad k = \frac{\pi}{2} \quad (2)$$

where T is the time for a cycle, k is a constant for making  $f_{NP}=1$  in the circular straining on  $\varepsilon-\gamma/\sqrt{3}$  plot and  $k=\pi/2$ .  $\varepsilon I(t)$  is the maximum absolute value of principle strain given by  $\varepsilon I(t)=\text{Max}[|\varepsilon_1(t)|, |\varepsilon_3(t)|]$  at time t and the  $\varepsilon I_{max}$  is the maximum value of  $\varepsilon I(t)$  in a cycle. In the equation, the angle  $\xi(t)/2$  is employed in order to describe the rotation of principal strain direction, Then,  $\xi(t)$  is the angle between  $\varepsilon I_{max}$  and  $\varepsilon I(t)$  and has double amplitude compared with that in the specimen [6]. The integrand measures the rotation of the maximum principal strain direction and the integration of strain amplitude after the rotation. Therefore,  $f_{NP}$  totally evaluates the severity of non-proportional straining in a cycle.

### Evaluation of Multiaxial LCF Life

To discuss the material dependence of life, this section shows multiaxial LCF properties of SUS316 and SGV410. Figures 4 (a) and (b) show  $N_f$  correlated by non-proportional strain range,  $\Delta\varepsilon_{NP}$ . In the figures, the bold line was drawn based on the data of Case 1 and the two thin lines show a factor of 2 band. The material constant,  $\alpha$ , employed here is determined by evaluating the degree of additional hardening. For SUS316 ( $\alpha=0.75$ ) in Fig. 4 (a),  $N_f$  in Case 2 is almost the same as that in Case 1. On the other hand,  $N_f$  in Case 2 for SGV410 ( $\alpha=0.39$ ) in Fig. 4 (b) is correlated unconservatively. The similar trend can also be observed in other FCC and BCC materials.

The difference of properties of reduction in life and additional hardening due to non-proportional loading may come from the difference of slip mechanism between FCC and BCC materials. Indeed, crack initiation and growth behaviors under

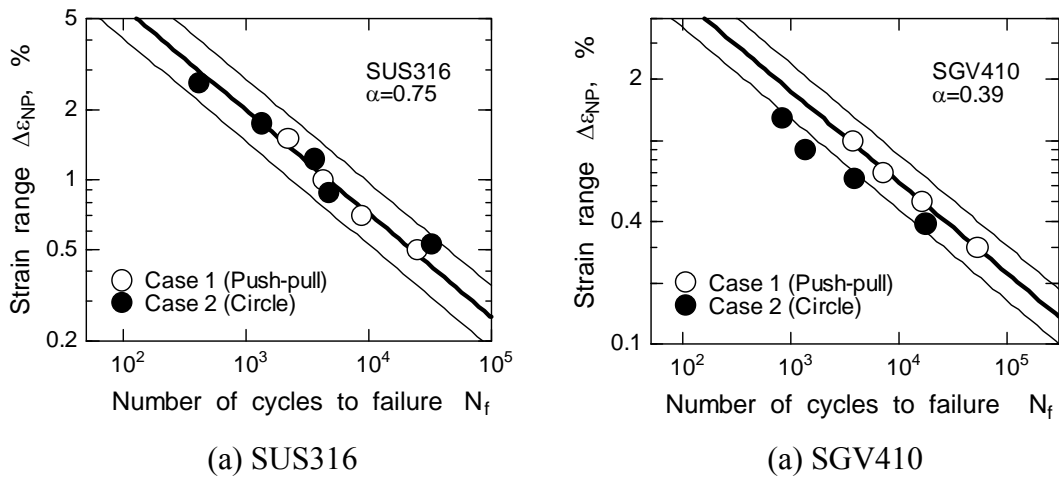


Figure 4. Relationship between  $\Delta\varepsilon_{NP}$  and  $N_f$ .

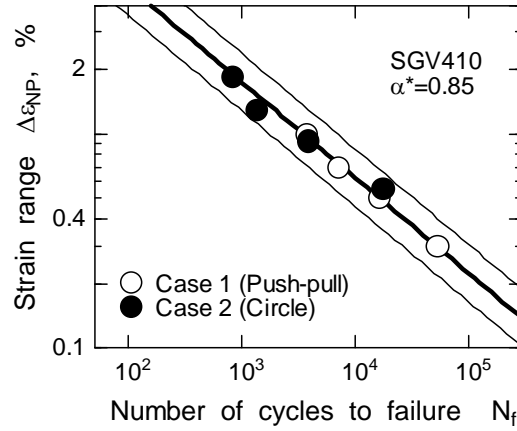


Figure 5. Relationship between  $\Delta\varepsilon_{NP}$  and  $N_f$  for SGV410 with  $\alpha^*=0.85$ .

proportional and non-proportional loadings are different, but more detail description which can be referred to previous Itoh's work [7] is omitted here due to a limitation of maximum number of page.

Figure 5 shows the re-plot of relationship between  $\Delta\varepsilon_{NP}$  and  $N_f$  for SGV410 by using  $\alpha^*$  as material constant for evaluating the degree of reduction in life. Correlation in Fig. 5 shows that  $N_f$  in Case 2 can be correlated within the factor of 2 band with  $\alpha^*=0.85$ .

#### ***Evaluation of Material Constant $\alpha$***

In order to investigate the relationship between properties of multiaxial LCF life and additional hardening under non-proportional loading, this section evaluates the relationship between the material constants  $\alpha$  and  $\alpha^*$  used in the non-proportional strain range,  $\Delta\varepsilon_{NP}$ , based on the results obtained by tested materials.

To obtain life curves with a small number of data in Case 1 and Case 2 for each material, the universal slope method was employed equated in Eq. 3 [11].

$$\Delta\varepsilon_{NP} = (1 + \alpha^* f_{NP}) \Delta\varepsilon I = AN_f^{-0.12} + BN_f^{-0.6} \quad (3)$$

where the coefficients  $A$  and  $B$  are equated as  $3.5\sigma_B/E$  and  $\varepsilon_f^{0.6}$ , respectively according to the definition of the universal slope method. Here,  $E$ ,  $\sigma_B$  and  $\varepsilon_f$  are Yong's modulus, strength and elongation. In this study,  $A$  is put as  $3.5\sigma_B/E$  accordingly, but  $B$  is determined as life curves based on the lives in Case 1.  $\alpha^*$  is put as lives in Case 1 and Case 2 correspond at the same  $\Delta\varepsilon I$  for each material. In this study, monotonic tension test were conducted for each material to obtain these material constants including 0.2% proof stress,  $\sigma_{0.2}$ . The constants obtained from the test were listed on Table 1.

Figure 6 shows the relationship between  $\alpha$  and  $\alpha^*$  for each material. The solid mark shows the data for BCC materials and the open mark for FCC materials. Keys shown in

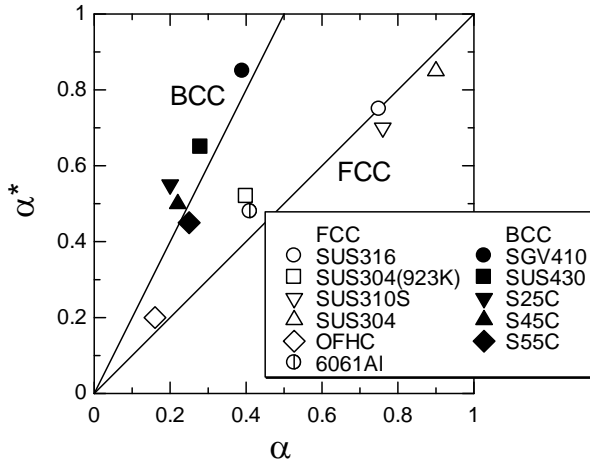


Figure 6. Relationship between  $\alpha$  and  $\alpha^*$ .

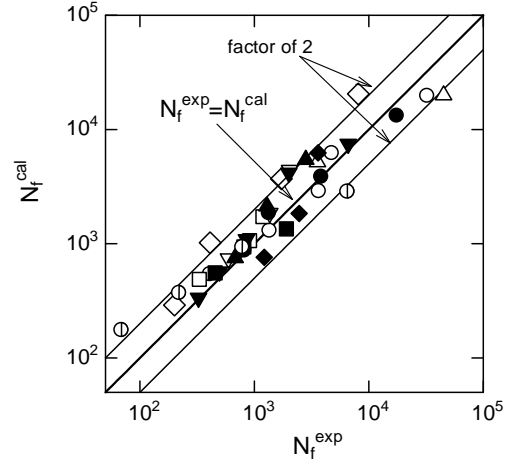


Figure 7. Comparison of  $N_f$  between calculation and experiment.

the figure will be referred in the following figures. The relationship is shown by two straight lines for FCC and BCC materials separately although a few data are scattered slightly. It suggests that reduction in life has closely relationship with additional hardening in non-proportional loading, which depends on crystal structure of tested materials. The relationship between  $\alpha$  and  $\alpha^*$  can be expressed experimentally as,

$$\alpha^* = \begin{cases} \alpha & \text{for FCC} \\ 2\alpha & \text{for BCC} \end{cases} \quad (4)$$

In order to verify the application of life estimation under non-proportional loading, the comparison of  $N_f^{\text{exp}}$  and  $N_f^{\text{cal}}$  in Case 2 is shown in Fig. 7.  $N_f^{\text{exp}}$  is the life obtained from experiment and  $N_f^{\text{cal}}$  the life evaluated by Eq. 1 based on life curve in push-pull test. In the calculation by Eq. 1,  $\alpha^*$  is used for material constant. All the data are correlated within the factor of 2 band, which suggests that failure lives under non-proportional loading for various materials can be estimated by  $\Delta\varepsilon_{\text{NP}}$  if the degree of additional hardening is known with using the relationship in Eq. 4.

## A SIMPLE METHOD FOR EVALUATION OF $\alpha$ AND LIFE EVALUATION

As discussed above, multiaxial LCF life shows the large reduction in life under non-proportional loading in comparison with that under proportional loading. By using non-proportional strain parameter,  $\Delta\varepsilon_{\text{NP}}$  in Eq. 1, multiaxial LCF life can be estimated from the data in push-pull loading test. However, to obtain the value of material constant,  $\alpha$ , multiaxial fatigue tests under non-proportional loading is necessary, but it is usually difficult to conduct. If  $\alpha$  can be obtained without conducting the multiaxial

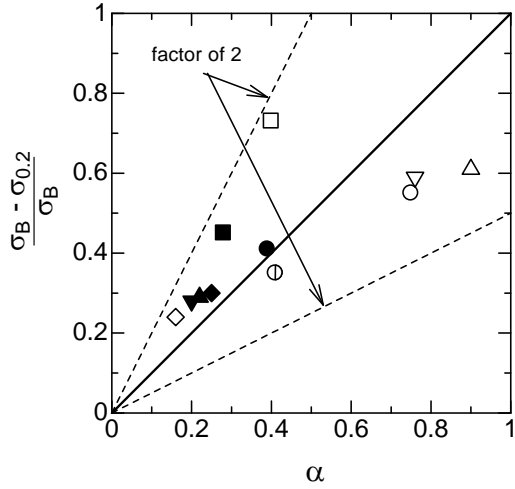


Figure 8. Relationship between  $(\sigma_B - \sigma_{0.2})/\sigma_B$  and  $\alpha$ .

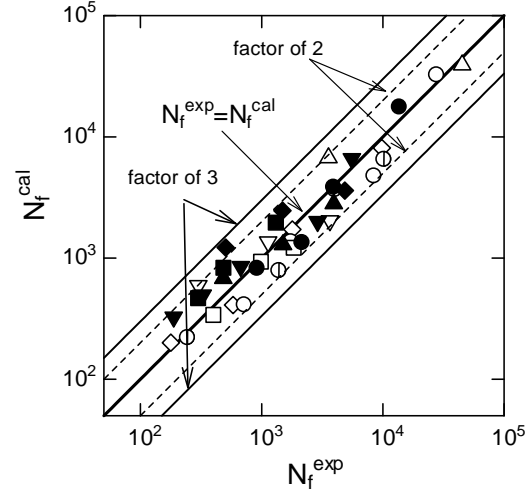


Figure 9. Comparison of  $N_f$  between experiment and calculation by Eq. 6.

fatigue test, it will be very convenient for engineers to estimate LCF life under non-proportional loading.

This section discusses the reevaluation of  $\alpha$  with focusing on the relationship between  $\alpha$  and material constants resulted from the monotonic tension test. Cyclic hardening and additional hardening behaviors should have close relationship with static deformation behavior, then relationship between  $(\sigma_B - \sigma_{0.2})/\sigma_B$  and  $\alpha$  is shown in Fig. 8. Although some scatter of data can be seen, but the relationship can be equated approximately as,

$$\alpha = \frac{(\sigma_B - \sigma_{0.2})}{\sigma_B} \quad (5)$$

According to Eqs 1, 4 and 5, non-proportional strain range,  $\Delta \varepsilon'_{NP}$ , can be rewritten as,

$$\Delta \varepsilon'_{NP} = \left( 1 + K \frac{\sigma_B - \sigma_{0.2}}{\sigma_B} f_{NP} \right) \Delta \varepsilon \quad (6)$$

where coefficient K takes K=1 for FCC materials and K=2 for BCC materials.

Figure 9 shows the comparison of  $N_f$  in Case 2 between experiment and calculation. In the figure,  $N_f^{\text{exp}}$  is the life in experiment and  $N_f^{\text{cal}}$  the life estimated based on life curve in push-pull test (Case 1 test) with using  $\Delta \varepsilon'_{NP}$  in Eq. 6. Consequently, all the data is correlated within the factor of 3 band and most of them correlated within the factor of 2 band. The good correlation in Fig. 9 indicates that multiaxial LCF life under non-proportional can be evaluated by Eq. (6) and material constants resulted from monotonic tension test.

## CONCLUSIONS

1. Reduction in life has closely relationship with additional hardening under non-proportional loading which depends on crystal structure of tested materials.
2.  $\alpha$  and  $\alpha^*$  which related to reduction in life and additional hardening due to non-proportional loading are reevaluated and can be equated by different linear relationships in BCC and FCC materials, respectively.
3.  $\alpha$  has a closely relation with the behavior of monotonic tension test and  $\alpha$  can be equated as  $\alpha = (\sigma_B - \sigma_{0.2}) / \sigma_B$ . The failure lives in non-proportional loading can be estimated by  $\Delta \varepsilon_{NP}$  where  $\alpha$  is replaced by  $(\sigma_B - \sigma_{0.2}) / \sigma_B$ .

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